Impact of source signal coloration on intensity vector based DOA estimation

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Abstract-An acoustic vector sensor provides measurements of both the pressure and particle velocity of a sound field in which it is placed. These measurements are vectorial in nature and can be used for the purpose of source localization. A straightforward approach towards determining the direction of arrival (DOA) utilizes the acoustic intensity vector, which is the product of pressure and particle velocity. The accuracy of an intensity vector based DOA estimator in the presence of sensor noise or reverberation has been analyzed previously for the case of a white source signal. In this paper, the effects of reverberation upon the accuracy of such a DOA estimator in the presence of a colored source signal are examined. The analysis is done with the aid of an extension to Polack's statistical room impulse response model which accounts for particle velocity as well as acoustic pressure. It is shown that signal colorations brings about a degradation in performance.

I. INTRODUCTION

Traditionally, direction of arrival (DOA) estimation is preformed with an array of pressure microphones. More recently, a device capable of supplying pressure and particle velocity measurements from a single location has become available [1].

Methods for DOA estimation utilizing these measurements were developed and measures of performance in the presence of sensor noise were proposed [2]. Analysis of performance in a reverberant environment (for the intensity vector method) demonstrated a certain inherent bias [3]. The goal of this paper is to asses the performance of an intensity vector based algorithm for determining DOA in a noiseless reverberant environment in the presence of a *colored* source signal.

This paper is organized as follows. Sec. II formulates the estimation problem and introduces relevant background and notation. Sec. III analyzes the expected estimation result for a given room impulse response (RIR) ensemble. In Sec. IV a statistical model of RIRs is presented which is useful for evaluating performance without specific knowledge of the RIR. Sec. V applies the statistical model for that purpose. In Sec. VI the scenario of a first order autoregressive (AR) source signal is used to demonstrate the negative impact of coloration and Sec. VI concludes with a brief summary.

II. PROBLEM FORMULATION AND PRELIMINARIES

In this section we formulate the problem, introduce the relevant notation and discuss the intensity vector method for DOA estimation. The true DOA will be represented as a unit-vector \mathbf{u} pointing from the sensor towards the source, and

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the estimate by a unit vector $\hat{\mathbf{u}}$. We wish to characterize any deviation which may arise between the two aforementioned orientations.

A single vector-sensor measures the pressure and particle velocity of a sound field at a given point in space. In [2], Nehorai and Paldi proposed an intensity vector based algorithm for estimating the DOA based on these measurements. Continuous-time intensity $i(\mathbf{r}, t)$ is the product of pressure and particle velocity:

$$\mathbf{i}(\mathbf{r},t) = p(\mathbf{r},t) \cdot \mathbf{v}(\mathbf{r},t), \qquad (1)$$

where **r** denotes the spatial location and *t* continuous-time. This vector corresponds to the magnitude and *direction* of the transport of acoustical energy [4], indicating its utility for determining DOA. Nehorai and Paldi assume that the signal behaves as a plane wave at the sensor location. Euler's linear equation $\rho_0 \frac{\partial}{\partial t} \mathbf{v}(\mathbf{r}, t) = -\nabla p(\mathbf{r}, t)$ implies that a plane-wave maintains the following equality:

$$\mathbf{v}(\mathbf{r},t) = \frac{-1}{\rho_0 c} p(\mathbf{r},t) \mathbf{u}, \qquad (2)$$

where ρ_0 is the ambient air density and c the speed of sound. Substituting the above into (1) yields an intensity vector $\mathbf{i}(\mathbf{r},t) = -\mathbf{u}p^2(\mathbf{r},t)/\rho_0 c$ in the opposite direction of the DOA. In order to eliminate the need to carry a constant coefficient throughout derivations, discrete-time particle velocity is taken as scaled: $\mathbf{v}[n] = [v_x[n] \ v_y[n] \ v_z[n]]^T = -\rho_0 c \mathbf{v}(\mathbf{r}_0, nT_s)$ [with T_s representing sampling-period and \mathbf{r}_0 the sensor location], such that:

$$\mathbf{i}[n] = p[n]\mathbf{v}[n] = p^2[n]\mathbf{u}.$$
(3)

Note, that the sampling of pressure measurements is conducted without scaling: $p[n] = p(\mathbf{r}_0, nT_s)$.

The intensity vector $\mathbf{i}[n]$ is not a completely reliable indicator of DOA, since it may be prone to random fluctuations. For example, at a given time instant n_0 , the sound arriving directly from the source may be negligible due to a momentary quietness, while the reverberative sound may be strong. In order to mitigate effects of random fluctuations upon estimation of \mathbf{u} , the intensity vector is averaged over a number of samples, and then $\hat{\mathbf{u}}$ is produced by normalization:

$$\overline{\mathbf{i}} = \frac{1}{N} \sum_{n=1}^{N} \mathbf{i}[n] = \frac{1}{N} \sum_{n=1}^{N} p[n] \mathbf{v}[n]$$
(4)

$$\hat{\mathbf{u}} = \frac{\mathbf{i}}{\|\mathbf{\bar{i}}\|} \,. \tag{5}$$

The accuracy of the estimation method can be evaluated by the *angular error* (AE) [2] which is defined as the angle δ by which $\hat{\mathbf{u}}$ deviates from the true DOA \mathbf{u} , or more formally:

$$\delta \equiv 2\sin^{-1}\left(\frac{\|\hat{\mathbf{u}} - \mathbf{u}\|}{2}\right) \,. \tag{6}$$

In analyzing the performance of the above algorithm in the presence of noise in *anechoic* environment, Nehorai and Paldi showed that under certain assumptions the DOA estimator $\hat{\mathbf{u}}$ is *unbiased* and *consistent* – converging almost surely (i.e., with probability 1) to the true value of \mathbf{u} as $N \to \infty$. Also, an asymptomatic measure of the rate at which the angular error δ converges to zero was derived.

In this paper, the absence of both internal (sensor) and external (ambient) noise is assumed. However, reverberation is present and may negatively impact DOA estimation.

The sound field $[p[n] \mathbf{v}[n]]^T$ at the sensor's location can be described as a source signal s[n] which is filtered by a set of RIRs. This is justified since the linear and timeinvariant properties of the wave equation apply equally to both pressure and particle velocity. The sound field can be therefore described as:

$$\begin{bmatrix} p[n] \\ v_x[n] \\ v_y[n] \\ v_z[n] \end{bmatrix} = \begin{bmatrix} (s*h_p)[n] \\ (s*h_{v_x})[n] \\ (s*h_{v_y})[n] \\ (s*h_{v_z})[n] \end{bmatrix} ,$$
(7)

or expressed more compactly: $[p[n] \mathbf{v}^T[n]]^T = (s * \mathbf{h})[n]$. The vector $\mathbf{h}[n] = [h_p[n] h_{v_x}[n] h_{v_y}[n] h_{v_z}[n]]^T$ consists of the room impulse responses pertaining to each sound field component measured. Subsequently, when referring to the entire RIR ensemble as opposed to particular coefficients, time dependency is dropped (i.e., \mathbf{h} is used instead of $\mathbf{h}[n]$).

The source signal is assumed to be wide sense stationary (WSS) process described by:

$$E\{s[n]\} = 0 \tag{8}$$

$$E\{s[n_1]s[n_2]\} = R_{ss}[n_1 - n_2].$$
(9)

In [3] the authors have shown that when distant intensity samples maintain an arbitrarily low correlation (i.e., $\lim_{\ell\to\infty} E\{\mathbf{i}^T[n]\mathbf{i}[n+\ell]\} = 0$), then \mathbf{i} , $\hat{\mathbf{u}}$ and δ will each converge *i.p.* to their respective *asymptotic* vales \mathbf{i}_{asym} , $\hat{\mathbf{u}}_{asym}$ and δ_{asym} . For the case of a white source signal (i.e., $R_{ss}[\ell] = \sigma^2 \delta[\ell]$), it was shown that $\hat{\mathbf{u}}_{asym}$ is biased, and quantitative terms characterizing this bias were derived. In the sequel, the effects of signal coloration are established.

III. ANALYSIS FOR A GIVEN RIR

In this section, we evaluate the expected outcome of the intensity based DOA estimator, assuming that the relevant RIR $\mathbf{h}[n] = [h_p[n] \mathbf{h}_v^T[n]]^T$ is given for all n. We start by inspecting the time-averaged intensity vector. Substitution of (7) into (4) produces:

$$\overline{\mathbf{i}} = \frac{1}{N} \sum_{n=1}^{N} (s * h_p)[n] \begin{bmatrix} (s * h_{v_x})[n] \\ (s * h_{v_y})[n] \\ (s * h_{v_z})[n] \end{bmatrix} .$$
(10)

The expected value of $\overline{\mathbf{i}}$ with respect to the signal s[n] (conditional upon the entire RIR ensemble h) is denoted $\psi(\mathbf{h})$ for brevity. Being that (10) represents a time invariant (although nonlinear) system and s[n] is WSS, it follows that all time instances of $\mathbf{i}[n]$ have identical distributions. Hence,

$$\psi(\mathbf{h}) \equiv E\{\mathbf{i}|\mathbf{h}\} = E\{\mathbf{i}[n]|\mathbf{h}\} = E\{p[n]\mathbf{v}[n]|\mathbf{h}\} = E\{(s * h_p)[n](s * \mathbf{h}_v)[n]|\mathbf{h}\} .$$
 (11)

Writing out the convolutions explicitly yields:

$$\psi(\mathbf{h}) = E\left\{\sum_{m_1} h_{\mathrm{p}}[m_1]s[n-m_1] \sum_{m_2} \mathbf{h}_v[m_2]s[n-m_2] \middle| \mathbf{h} \right\}$$
$$= \sum_{m_1} \sum_{m_2} E\{s[n-m_1]s[n-m_2]\}h_{\mathrm{p}}[m_1]\mathbf{h}_v[m_2]$$
$$= \sum_{m_1} \sum_{m_2} R_{ss}[m_2-m_1]h_{\mathrm{p}}[m_1]\mathbf{h}_v[m_2], \qquad (12)$$

which is independent of the time index n. After substitution of variables $\ell = m_2 - m_1$ and $m = m_1$, (12) yields:

$$\psi(\mathbf{h}) = \sum_{\ell} R_{ss}[\ell] \sum_{m} h_p[m] \mathbf{h}_v[m+\ell].$$
(13)

We adopt the notation that the direct arrival corresponds to $\mathbf{h}[-n_d]$ for some positive n_d and the reflections correspond to $\mathbf{h}[n \ge 0]$.¹ Then, the expected intensity vector can be represented as:

$$\psi(\mathbf{h}) = R_{ss}[0]h_p[-n_d]\mathbf{h}_v[-n_d] + \sum_{[\ell,m]\in\mathbb{Z}^2\setminus[0,-n_d]}R_{ss}[\ell]h_p[m]\mathbf{h}_v[m+\ell], \quad (14)$$

with \mathbb{Z}^2 representing the set of ordered integer pairs. The first term on the right-hand side corresponds to the direct arrival and is oriented towards the true DOA. The second term contains effects of reverberation and may cause estimation error. Since the true DOA is contained in $h_p[-n_p]\mathbf{h}_v[n]$ and the $\hat{\mathbf{u}}_{asym}$ is given by normalization of (13), it follows that δ_{asym} is uniquely determined by \mathbf{h} and $R_{ss}[\ell]$. It is of particular interest to characterize the second term of the righthand side of (14) in order to gain insight into the magnitude of the angular error (AE).

¹Obviously this cannot be literally accurate since the direct arrival would defy causality. Rather, it should be viewed as a time shifted version of the RIR such that the reflections start at n = 0. The benefits of this notation are twofold. Firstly, reverberation starting at time n = 0 is less cumbersome to manipulate mathematically. Secondly, any complications arising from the propagation delay of the direct arrival being a fraction of the sampling rate are circumvented.

IV. STATISTICAL MODEL FOR RIR ENSEMBLE

As the number of available samples N grows, $\overline{\mathbf{i}}$ asymptotically approaches its expected value, given by (13) providing the orientation of the estimated DOA. In practice this fact cannot be used *directly* since the RIRs are generally unknown. In fact, if they were to be known then the true DOA could be extracted from $h[-n_d]$ and no estimation would be necessary. Often, general properties describing the behavior of RIRs such as the decay time RT_{60} are available. The field of *statistical* room acoustics (SRA) uses these properties to systematically characterize reverberation by employing probabilistic tools. Polack (in his thesis written in French; see Jot et al. [5] for a survey in English) utilizes the diffuse nature of reverberation in which many reflections arrive simultaneously to describe the pressure RIR $h_p[n]$ as an independent, identically distributed (i.i.d.) Gaussian process with zero-mean multiplied by an exponentially decaying envelope.

A new model has been suggested [3] which extends Polack's model to incorporate particle velocity. The RIRs are presented as:

$$\mathbf{h}[n] \equiv \begin{bmatrix} h_{\mathbf{p}}[n] \\ h_{v_x}[n] \\ h_{v_y}[n] \\ h_{v_z}[n] \end{bmatrix} = A_d \begin{bmatrix} 1 \\ \mathbf{u} \end{bmatrix} \delta[n+n_d]$$
(15)
$$+ \sigma_0 \begin{bmatrix} w_1[n] \\ 1/\sqrt{3} & w_2[n] \\ 1/\sqrt{3} & w_3[n] \\ 1/\sqrt{3} & w_4[n] \end{bmatrix} \epsilon[n] e^{-\alpha n} ,$$

where A_d corresponds to the amplitude of the direct arrival coefficients and σ_0 to reverberation amplitude, $\epsilon[n]$ is the discrete-time unit step function and $[w_1[n] \ w_2[n] \ w_3[n] \ w_4[n]]^T = \mathbf{w}[n]$ is an i.i.d. Gaussian process, such that:

$$E\{\mathbf{w}[n]\} = \mathbf{0} \tag{16a}$$

$$E\{\mathbf{w}[n_1]\mathbf{w}^T[n_2]\} = \mathbf{I}_4 \cdot \delta[n_1 - n_2], \qquad (16b)$$

where I_K represents a $K \times K$ identity matrix. The envelope decay parameter α is linked to the reverberation time RT_{60} by the following relationship [5]:

$$\mathbf{RT}_{60} = \frac{3\ln(10)}{\alpha f_{\rm s}} \,, \tag{17}$$

with $f_s = 1/T_s$ denoting the sampling frequency.

The basis for the statistical relationships predicated by the model is founded upon the covariance properties [6], [7] of the pressure and particle velocity components of a point situated in an ideally diffuse and isotropic sound field. This model, further elaborated upon in [3], proves useful in analysis of typical estimation error.

Strictly speaking, reverberation is a deterministic process being governed by the wave equation and the physical boundary conditions of its environment. Nonetheless, these processes are extremely complex and are more conveniently described by SRA as random, or more precisely *pseudorandom* [since the environment and source-receiver positions are actually fixed]. The operations of expectation and variance which are used in the sequel with respect to **h** may be viewed as spatial averaging over different possible setups in the environment and for clarity are henceforth denoted $E_s\{\circ\}$ and $Var_s\{\circ\}$, respectively.

V. ANALYSIS OF ESTIMATOR WITH PROPOSED SRA MODEL

Substituting (15) into (12), and applying spatial expectation yields:

$$E_{s}\{\psi(\mathbf{h})\} = R_{ss}[0]h_{p}[-n_{d}]\mathbf{h}_{v}[-n_{d}] = A_{d}^{2}R_{ss}[0]\mathbf{u}.$$
 (18)

Similar substitution for the second moment yields:

$$E_{s}\{\psi(\mathbf{h})\psi^{T}(\mathbf{h})\} = E_{s}\left\{\sum_{m_{1}}\sum_{m_{2}}R_{ss}[m_{2}-m_{1}]h_{p}[m_{1}]\mathbf{h}_{v}[m_{2}] \times \sum_{m_{3}}\sum_{m_{4}}R_{ss}[m_{4}-m_{3}]h_{p}[m_{3}]\mathbf{h}_{v}^{T}[m_{4}]\right\}$$
$$=\sum_{m_{1}}\sum_{m_{2}}\sum_{m_{3}}\sum_{m_{4}}\left(R_{ss}[m_{2}-m_{1}]R_{ss}[m_{4}-m_{3}] \times E_{s}\{h_{p}[m_{1}]\mathbf{h}_{v}^{T}[m_{2}]h_{p}[m_{3}]\mathbf{h}_{v}[m_{4}]\}\right).$$
(19)

The expression $E_s\{h_p[m_1]\mathbf{h}_v^T[m_2]h_p[m_3]\mathbf{h}_v[m_4]\}$ takes the expectation of the product of four terms which are Gaussian variables or constants (when corresponding to the direct arrival at $-n_d$). After applying the formula for high-order Gaussian moments, nonzero terms only appear when $m_1 = m_3$ and $m_2 = m_4$. Thus,

$$E_{s}\{\psi(\mathbf{h})\psi^{T}(\mathbf{h})\}$$

$$= \sum_{m_{1}} \sum_{m_{2}} R_{ss}^{2}[m_{2} - m_{1}]E_{s}\{h_{p}^{2}[m_{1}]\}E_{s}\{\mathbf{h}_{v}[m_{2}]\mathbf{h}_{v}^{T}[m_{2}]\}.$$
(20)

This expression can be split into components resulting from the different possible combinations of direct and reverberant products:

$$E_{\rm s}\{\psi(\mathbf{h})\psi^T(\mathbf{h})\}\tag{21}$$

$$=\sum_{m_1=0}^{\infty}\sum_{m_2=0}^{\infty}R_{ss}^2[m_2-m_1]E_{s}\{h_p^2[m_1]\}E_{s}\{\mathbf{h}_v[m_2]\mathbf{h}_v^T[m_2]\}$$
(i)

+
$$\sum_{m_2=0}^{\infty} R_{ss}^2 [m_2 + n_d] h_p^2 [-n_d] E_s \{ \mathbf{h}_v [m_2] \mathbf{h}_v^T [m_2] \}$$
 (ii)

+
$$\sum_{m_1=0}^{\infty} R_{ss}^2 [-n_d - m_1] E_{s} \{h_p^2[m_1]\} \mathbf{h}_v[-n_d] \mathbf{h}_v^T[-n_d]$$
 (iii)

$$+ R_{ss}^{2}[0]h_{p}^{2}[-n_{d}]\mathbf{h}_{v}[-n_{d}]\mathbf{h}_{v}^{T}[-n_{d}].$$
 (iv)

Explicit substitution of (15) produces:

$$E_{\rm s}\{\psi(\mathbf{h})\psi^{T}(\mathbf{h})\} =$$

$$\mathbf{I}_{\rm s} \begin{bmatrix} \mathbf{1}_{\sigma} \mathbf{1} & \sum_{k=1}^{\infty} B^{2} \left[\ell\right]_{\sigma} - 2\alpha|\ell|$$
(22)

$$\frac{13}{3} \frac{1}{3} \frac{1}{1} \frac{1$$

+
$$\mathbf{I}_3 \cdot \frac{1}{3} \sigma_0^2 A_d^2 \sum_{\ell=0} R_{ss}^2 [\ell + n_d] e^{-2\alpha \ell}$$
 (ii)

$$+\mathbf{u}\mathbf{u}^{T}\cdot\sigma_{0}^{2}A_{d}^{2}\sum_{\ell=0}^{\infty}R_{ss}^{2}[\ell+n_{d}]e^{-2\alpha\ell}$$
(iii)

$$+ \mathbf{u}\mathbf{u}^T A_d^4.$$
 (iv)

Comparing term (iv) with (18) demonstrates that (iv) = $E_{s}\{\psi(\mathbf{h})\}E_{s}^{T}\{\psi(\mathbf{h})\}$. Hence,

$$Var_{s}{\psi(\mathbf{h})} = (i) + (ii) + (iii).$$
 (23)

Interpretation of the above analysis is as follows. From (18) we conclude that the spatial mean of the asymptotic estimator points towards **u** and is consequently unbiased. However a *particular spatial instance* (i.e., source receiver setup) will likely deviate from this ideal as the variance (23) is nonzero. Terms (i) and (ii) manifest equal variance in all direction (as indicated by the identity matrix), whereas term (iii) runs parallel to the true DOA. Therefore, the former two terms can be expected to have a more dominant effect on the estimation error. It should also be noted that terms (ii) and (iii) which are affected by the direct arrival A_d do not contain a term corresponding to $R_{ss}[0]$ and thus will vanish in the case of a white source signal.

VI. NUMERICAL RESULTS

We proceed to examine the case of noise produced by a first order AR process s[n] with autocorrelation $R_{ss}[\ell] = e^{-\beta|\ell|}$. Consequently, the variance becomes:

$$\operatorname{Var}_{s}\{\psi(\mathbf{h})\} =$$
(24)

$$\mathbf{I}_{3} \cdot \frac{1}{3} \sigma_{0}^{4} \frac{1}{1 - e^{-4\alpha}} \left(1 + 2 \frac{e^{-2(\alpha + \beta)}}{1 - e^{-2(\alpha + \beta)}} \right)$$
(i)

+
$$\mathbf{I}_3 \cdot \frac{1}{3} \sigma_0^2 A_d^2 \frac{e^{-2(\alpha+\beta)n_d}}{1 - e^{-2(\alpha+\beta)}}$$
 (ii)

$$+\mathbf{u}\mathbf{u}^{T}\cdot\sigma_{0}^{2}A_{d}^{2}\frac{e^{-2(\alpha+\beta)n_{d}}}{1-e^{-2(\alpha+\beta)}}.$$
 (iii)

A plot of the square root of the three terms which constitute $\operatorname{Var}_{s}\{\psi(\mathbf{h})\}\$ as a function of autocorrelation is demonstrated in Fig. 1(a). In this particular example RT_{60} was set to 0.5 s. To enable comparison of reverberation and source signal correlation on the same scale, the correlation is measured in units of decay time (DT) which is related to β in a fashion analogous to the association between RT_{60} and α [see (17)], specifically: $\operatorname{DT} = \frac{3\ln(10)}{\beta f_s}$. Thus, inspection of the plot indicates the impact of source coloration even when correlation decays much faster than reverberation $\operatorname{DT} \ll \operatorname{RT}_{60}$. The parameters A_d and σ_0 were chosen such that the direct to reverberation ratio (DRR) which is defined as $\operatorname{DRR} = h_p^2 [-n_d] / \sum_{n=0}^{\infty} h_p^2[n]$



(a) Square root of intensity vector (b) Mean asymptotic angular error. variance terms.

Fig. 1. Square root of intensity vector variance terms and the corresponding angular error as a function of intersignal correlation.

receives a value of 2 (\approx 3 dB), n_d was set to 1 and f_s to 8 kHz. The y-axis which is the square root of variance terms has units akin to standard deviation. Note that for DT = 0 s (i.e., a white souse signal), term (i) is the only nonzero component.

The graph exhibits a significant increase in variance for higher autocorrelation levels. This indicates a corresponding increase in estimation error. Fig. 1(b) shows the Monte-Carlo results from 100 RIR ensembles generated according to (15) [and truncated after 1.5 s] with parameters identical to those described in the previous paragraph. The asymptotic intensity average is then computed by (13), and the AE calculated from (6). The sample averaged AE is displayed (in degrees) as a function of DT. The graph bears out the prediction that higher autocorrelation levels negatively affect estimation accuracy.

VII. CONCLUSION

The statistical properties of the average intensity vector were analyzed. Based on SRA theory, an analytic expression for the variance of the expected intensity vector was derived. This variance predicts the behavior of the AE as a function of the decay time of the source signal. It was then empirically verified that increased signal autocorrelation escalates intensity variance and DOA estimation is significantly degraded.

REFERENCES

- H.-E. de Bree, P. Leussink, I. T. Korthorst, D. H. Jansen, D. T. Lammerink, and P. M. Elwenspoek, "The microflown, a novel device measuring acoustical flows," in 8th International Conference on Solid-State Sensors and Actuators, 1995 and Eurosensors IX. Transducers 1995, vol. 1. IEEE, 1995, pp. 536–539.
- [2] A. Nehorai and E. Paldi, "Acoustic vector-sensor array processing," Signal Processing, IEEE Transactions on, vol. 42, no. 9, pp. 2481–2491, Sep 1994.
- [3] D. Levin, E. Habets, and S. Gannot, "On the angular error of intensity vector based DOA estimation in reverberant sound fields," conditionally accepted for publication in *J. Acoust. Soc. Amer.*, Jun. 2010.
- [4] A.D. Pierce, Acoustics An Introduction to its physical principles and applications. Acoustical Society of America, 1991.
- [5] J. Jot, L. Cerveau, and O. Warusfel, "Analysis and synthesis of room reverberation based on a statistical time-frequency model," *Preprint No.* 4629 of the 103th Audio Engineering Society Convention, 1997.
- [6] F. Jacobsen and T. Roisin, "The coherence of reverberant sound fields," J. Acoust. Soc. Amer., vol. 108, no. 1, pp. 204–210, 2000.
- [7] M. Hawkes and A. Nehorai, "Acoustic vector-sensor correlations in ambient noise," *Oceanic Engineering, IEEE Journal of*, vol. 26, no. 3, pp. 337–347, Jul 2001.