# MIMO INSTANTANEOUS BLIND IDENTIFICATION BASED ON SECOND ORDER TEMPORAL STRUCTURE AND HOMOTOPY METHOD

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**ABSTRACT**

Blind identification is of paramount importance for well-known signal processing problems such as Blind Signal Separation and Direction Of Arrival (DOA) estimation. This paper presents a new method for *Multiple-Input Multiple-Output Instantaneous Blind Identification* based on second order temporal statistical variabilities in the data, such as non-whiteness and non-stationarity. The method basically consists of two stages. Firstly, based on certain assumptions about the statistical structure and diversity of the signals and system, and using subspace techniques, the problem is formulated in such a way that each column of the mixing matrix satisfies a system of multivariate homogeneous polynomial equations [1–3]. Then, this nonlinear system is solved by means of a so-called homotopy method. Homotopy methods provide a general tool for solving systems of nonlinear equations by smoothly deforming the known solutions of a simple start system into the desired solutions of the target system [4, 5]. Our two-stage identification approach allows to estimate more sources than sensors, something that is often believed to be impossible with second order statistics. The method is applied to the Instantaneous Blind Signal Separation of speech signals [6].

## 1. INTRODUCTION

In this paper, we consider the so-called *Multiple-Input Multiple-Output (MIMO) Instantaneous Blind Identification* problem, which we will briefly denote by the acronym *MIBI*. In this problem, a number of mutually statistically independent source signals are mixed by a MIMO instantaneous mixing system and only the mixed signals are available, i.e. both the mixing system and the original source signals are unknown or *blind* [6]. The goal of MIBI is to recover the instantaneous MIMO system, or its parameters (e.g. in the case of DOA estimation), from the observed mixtures of the source signals only. Fig. 1 (see next page) shows the MIBI problem setup for $S$ source and $D$ sensor signals. The source, sensor and additive noise signals are denoted by $s_1[n], \ldots, s_S[n]$, $x_1[n], \ldots, x_D[n]$ and $\nu_1[n], \ldots, \nu_D[n]$ respectively. The instantaneous mixing system is modeled by a matrix $\mathbf{A}$ of size $D \times S$. A problem closely related to MIBI is *Instantaneous Blind Signal Separation (IBSS)* [6], which deals with the problem of separating mutually statistically independent sources from their observed instantaneous mixtures only. Contrary to MIBI, the main interest in IBSS is in the source signals instead of the mixing system. In fact, once MIBI has been performed, the source signals can be recovered (approximately) by applying the (pseudo-)inverse of the estimated mixing system to the observed mixtures. In this paper, the main focus will be on MIBI, while IBSS will be considered as an application.

It is widely recognized that many possible applications exist for MIBI and IBSS [6]. Examples of (parameterized) MIBI can be found in source localization problems, which are very important in many sensor array systems, such as radar and sonar. Several examples of IBSS can be found in the field of biomedical engineering, where the goal of several applications is to reveal independent sources in different kinds of biological signals like EEG's and ECG's. Other examples can be found in the separation of speech signals, images, etc. Although many practical problems can be described more adequately by more complex MIMO Blind Identification models such as convolutive and/or non-linear ones, MIBI can often be used as a good starting point, e.g. for a frequency domain approach in the convolutive case.

In previous publications [2, 3], we presented a unifying view at MIBI based on exploiting the temporal structure in the data of some arbitrary fixed order. It was shown that, under certain assumptions, employing $l$-th order statistics and $D$ sensors, a system of $D$-variate $l$-homogeneous polynomial equations (see Section 3) could be derived, the solutions of which are given by the columns of the mixing matrix. This problem formulation in a natural way allows to deal with the practically important case of more sources than sensors ($S > D$), even in the case of Second Order Statistics (SOS). Especially in DOA estimation, which is a parameterized version of MIBI, the number of source DOA's that can be estimated may greatly exceed the number of sensors [3]. In [1], a practical algorithm based on SOS and the Generalized Eigenvalue Decomposition (GEVD) was given for the 'square MIBI' case with $D = S$. Likewise, in [3] a MUSIC-like spatial pseudo-spectrum exhibiting sharp peaks at the source DOA's was computed. In [1–3] the main focus was on providing algebraic and geometric insight into the derivation and properties of the system of polynomial equations. For the general MIBI case, we did not yet provide an algorithm that can solve the system of equations for $S > D$. The main purpose of this paper is to provide a MIBI method that is based on SOS, uses a homotopy method, is able to handle both the $S \leq D$ and $S > D$ cases, and is formulated in such a way that it can easily be generalized to more general scenarios with arbitrary order statistics.

The outline of the paper is as follows. Firstly, the structure and assumptions of the MIBI model are explained in Section 2, along with some notation. After that, the derivation of the system of homogeneous polynomial equations is briefly recapitulated in Section 3, and some of its nice algebraic and geometric properties are highlighted. Then, in Section 4 we present a homotopy method for solving the system. Subsequently, in Section 5 the theory is applied to an example of a MIBI scenario with $D = 3$ sensors and $S = 4$ sources, which demonstrates the ability of handling more sources than sensors with SOS only. Finally, conclusions and future research are discussed in Section 6.

## 2. STRUCTURE AND ASSUMPTIONS OF MIBI MODEL

A block diagram of the MIBI problem setup is shown in Fig. 1. A set $\{s_1[n], \ldots, s_S[n]\}$ of $S$ mutually statistically independent source signals is mixed by a MIMO linear instantaneous mixing system that is represented by a matrix $\mathbf{A}$, and only a set $\{x_1[n], \ldots, x_D[n]\}$ of $D$ sensor signals corrupted by $D$ additive noise signals in the set $\{\nu_1[n], \ldots, \nu_D[n]\}$ is available. Both the signals and the system are assumed to be real-valued. Mathematically, the MIBI observation model can be written as follows:

$$\mathbf{x}[n] = \sum_{j=1}^{S} \mathbf{a}^j \, s_j[n] + \boldsymbol{\nu}[n] = \mathbf{A}\mathbf{s}[n] + \boldsymbol{\nu}[n] \quad \forall\, n \in \mathbb{Z}, \tag{2.1}$$

where

$$\mathbf{x}[n] \triangleq \begin{bmatrix} x_1[n] \\ \vdots \\ x_D[n] \end{bmatrix}, \; \mathbf{s}[n] \triangleq \begin{bmatrix} s_1[n] \\ \vdots \\ s_S[n] \end{bmatrix}, \; \boldsymbol{\nu}[n] \triangleq \begin{bmatrix} \nu_1[n] \\ \vdots \\ \nu_D[n] \end{bmatrix}, \; \mathbf{a}^j \triangleq \begin{bmatrix} a_1^j \\ \vdots \\ a_D^j \end{bmatrix}$$

are column vectors of sensor signals, source signals, additive noise signals and mixing elements respectively. Subscript and superscript indices are used to index the components of a column and row vector respectively. Furthermore, the symbol $n$ denotes discrete time. The coefficient $a_i^j$ denotes the instantaneous transfer from the $j$-th source to the $i$-th sensor. From (2.1), it easily follows that two *indeterminacies* are inherent to the MIBI model [6], viz. the scalings/norms (including signs) and the order of the columns of the mixing matrix cannot be resolved. In general, they do not cause serious problems because for many applications the most relevant information is contained in the 'directions' of the columns or the waveforms of the source signals, rather than in their magnitudes or order.

Our derivation of the system of polynomial equations satisfied by the vectors $\mathbf{a}^1, \ldots, \mathbf{a}^S$ is based on several *assumptions which ensure that sufficient temporal and spatial diversity is present*. We express them mathematically in terms of the auto- and cross-correlation functions of the source and noise signals for different times $n$ (allowing the exploitation of (block-wise) non-stationarity) and/or different lags $m$ (allowing the exploitation of non-whiteness):

*AS1:* $r^s_{j_1 j_2}[n;m] \triangleq E\{s_{j_1}[n]\, s_{j_2}[n-m]\} = 0 \quad \forall\, n, m \in \mathbb{Z},$
$m \neq 0, \;\; 1 \leq j_1 \neq j_2 \leq S;$

*AS2:* $\sum_{j=1}^{S} \xi^j \, r^s_{jj}[n;m] = 0 \quad \forall\, n \in \mathbb{Z},\; m \neq 0 \quad \Longrightarrow$
$\xi^j = 0 \quad \forall\, 1 \leq j \leq S;$

*AS3:* $r^\nu_{i_1 i_2}[n;m] \triangleq E\{\nu_{i_1}[n]\, \nu_{i_2}[n-m]\} = \left(\sigma^\nu[n]\right)^2 \delta[m]$
$\forall\, n, m \in \mathbb{Z}, \;\; 1 \leq i_1, i_2 \leq D;$

*AS4:* $r^{s\nu}_{ji}[n;m] \triangleq E\{s_j[n]\, \nu_i[n-m]\} = 0 \quad \forall\, n, m \in \mathbb{Z},$
$m \neq 0, \;\; 1 \leq i \leq D, \;\; 1 \leq j \leq S.$

Note that *AS2* states that the source auto-correlation functions are linearly independent for $m \neq 0$. The expression $\left(\sigma^\nu[n]\right)^2$ in *AS3* denotes the (possibly time-dependent) variance of the noise signals, and $\delta[m]$ denotes the Kronecker delta function. We assume that the involved correlations functions, which are defined in terms of the mathematical expectation operator $E\{\cdot\}$, are (block-)ergodic and can be estimated 'sufficiently accurate' by means of time-averaging over certain blocks of data. In the sequel, all correlation functions are considered on a certain Region Of Support (ROS) $\mathcal{T} \triangleq \left[[n_1; m_1], \ldots, [n_N; m_N]\right]$, where $[n;m]$ pairs with $m = 0$ are excluded. Note that such a ROS is 'noise-free' for white noise (see *AS3*). Each correlation function is represented by a row vector containing $N \geq S$ (estimated) values. For example, the cross-correlation function between the $i_1$-th and $i_2$-th sensor signals is represented by the correlation row vector $\tilde{\mathbf{r}}^x_{i_1 i_2} \triangleq \left[ r^x_{i_1 i_2}[n_1; m_1] \;\; \cdots \;\; r^x_{i_1 i_2}[n_N; m_N] \right]$. As is justified in [3], all assumptions made above are quite reasonable in several practical applications.

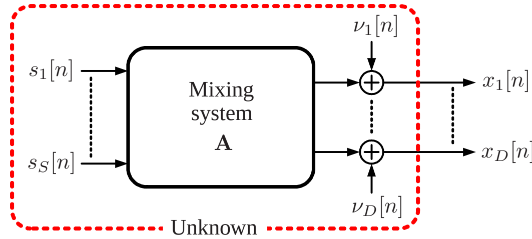


Figure 1: Block diagram of MIBI problem setup.

## 3. SYSTEM OF EQUATIONS FOR SOS

In this section, the derivation of the system of homogeneous polynomial equations is briefly recapitulated. A more elaborate derivation for more general scenarios and an analysis of the algebraic and geometric properties of the system can be found in [2, 3]. We start the derivation by expressing the sensor correlation functions in the source and noise correlation functions. Using *AS1, AS3* and *AS4*, the sensor correlation function $r^x_{i_1 i_2}[n_1, n_2]$ of the $i_1$-th and $i_2$-th sensor signals can be expressed as follows for all $1 \leq i_1, i_2 \leq D$:

$$r^x_{i_1 i_2}[n;m] = \sum\nolimits_{j=1}^{2} a^j_{i_1} a^j_{i_2}\, r^s_{jj}[n;m] \quad \forall\, [n;m] \in \mathcal{T}. \tag{3.1}$$

This equation implies that $r^x_{i_1 i_2}[n;m] = r^x_{i_2 i_1}[n;m]$ for all $[n;m] \in \mathcal{T}$ and $1 \leq i_1, i_2 \leq D$. Hence, the sensor correlation functions and associated row vectors are only 'essentially different' for the set of index pairs:

$$\mathcal{I}^{\mathrm{a}}_{2,D} \triangleq \left\{(i_1, i_2) \mid 1 \leq i_1, i_2 \leq D\right\}, \tag{3.2}$$

with cardinality $\left|\mathcal{I}^{\mathrm{a}}_{2,D}\right| = \frac{1}{2} D(D+1)$ (the subscript 'a' stands for 'ascending'). In the sequel, the set containing only the essentially different sensor correlation row vectors $\tilde{\mathbf{r}}^x_{i_1 i_2}$, i.e. the ones with $(i_1, i_2) \in \mathcal{I}^{\mathrm{a}}_{2,D}$, is denoted by $\mathcal{K}^x_D$:

$$\mathcal{K}^x_D \triangleq \left\{ \tilde{\mathbf{r}}^x_{i_1 i_2} \mid (i_1, i_2) \in \mathcal{I}^{\mathrm{a}}_{2,D} \right\}. \tag{3.3}$$

By (3.1), each vector in this set can be expressed as a linear combination of the source auto-correlation vectors:

$$\tilde{\mathbf{r}}^x_{i_1 i_2} = \sum\nolimits_{j=1}^{2} a^j_{i_1} a^j_{i_2}\, \tilde{\mathbf{r}}^s_{jj}\,. \tag{3.4}$$

Due to *AS2* the dimension $d^s$ of the linear space $\mathcal{L}\left(\{\tilde{\mathbf{r}}^s_{jj}\}_{1 \leq j \leq S}\right)$ spanned by the source auto-correlation row vectors equals the number of sources $S$. Hence, from (3.4) it follows that the dimension $d^x$ of the linear space $\mathcal{L}\left(\mathcal{K}^x_D\right)$ spanned by the sensor correlation row vectors is smaller than or equal to $d^s$, i.e. $d^x \leq S$. Consequently, *the sensor correlation vectors (and functions) are linearly dependent whenever* $\left|\mathcal{K}^x_D\right| > S$. In all subsequent derivations, we will assume this explicitly:

$$\left|\mathcal{K}^x_D\right| = \left|\mathcal{I}^{\mathrm{a}}_{2,D}\right| = \frac{1}{2} D(D+1) > S\,. \tag{3.5}$$

Hence, by the definition of linear dependence there exist nonzero and non-unique sets of coefficients $\left\{\varphi^{i_1 i_2}_q\right\}^{(i_1,i_2) \in \mathcal{I}^{\mathrm{a}}_{2,D}}$ indexed by an integer-valued index $q$ such that:

$$\sum\nolimits_{(i_1,i_2) \in \mathcal{I}^{\mathrm{a}}_{2,D}} \varphi^{i_1 i_2}_q \, \tilde{\mathbf{r}}^x_{i_1 i_2} = \tilde{\mathbf{0}} \quad \forall\, q \in \mathcal{Q}, \tag{3.6}$$

where $\mathcal{Q} \triangleq \{1, \ldots, Q\}$ and $Q \triangleq |\mathcal{Q}|$ is the maximum number of linearly independent equations that will be determined soon. Defining the matrices $\mathbf{\Phi}$ and $\mathbf{C}^x$ as follows:

$$\mathbf{\Phi} \triangleq \begin{bmatrix} \tilde{\boldsymbol{\varphi}}_1 \\ \vdots \\ \tilde{\boldsymbol{\varphi}}_Q \end{bmatrix} = \begin{bmatrix} \varphi_1^{11} & \varphi_1^{12} & \cdots & \varphi_1^{DD} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_Q^{11} & \varphi_Q^{12} & \cdots & \varphi_Q^{DD} \end{bmatrix} \tag{3.7}$$

and

$$\mathbf{C}^x \triangleq \begin{bmatrix} \tilde{\mathbf{r}}_{11}^x \\ \tilde{\mathbf{r}}_{12}^x \\ \vdots \\ \tilde{\mathbf{r}}_{DD}^x \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{r}}_{11}^x[n_1; m_1] & \cdots & \tilde{\mathbf{r}}_{11}^x[n_N; m_N] \\ \tilde{\mathbf{r}}_{12}^x[n_1; m_1] & \cdots & \tilde{\mathbf{r}}_{12}^x[n_N; m_N] \\ \vdots & \ddots & \vdots \\ \tilde{\mathbf{r}}_{DD}^x[n_1; m_1] & \cdots & \tilde{\mathbf{r}}_{DD}^x[n_N; m_N] \end{bmatrix}, \tag{3.8}$$

system (3.6) can now simply be written as:

$$\mathbf{\Phi}\,\mathbf{C}^x = \mathbf{0}_Q^N\,, \tag{3.9}$$

where $\mathbf{0}_Q^N$ denotes the zero matrix with $Q$ rows and $N$ columns. This shows that the rows of $\mathbf{\Phi}$ lie in the left null space of $\mathbf{C}^x$, which can easily be determined from the SVD of $\mathbf{C}^x$ by choosing the left singular vectors that correspond to the smallest singular values. Since $\mathrm{rank}(\mathbf{C}^x) = \dim\big(\mathcal{L}\,(\mathcal{K}_D^x)\,\big) = d^x$, the dimension $Q$ of the left null space equals $\left|\mathcal{K}_D^x\right| - d^x = \frac{1}{2}D(D+1) - d^x$. Now we show how these observations can be used to derive a system of multivariate homogeneous second degree polynomial equations satisfied by each column of the mixing matrix $\mathbf{A} = \left[\mathbf{a}^1 \ \cdots \ \mathbf{a}^S\right]$. Substituting (3.4) into (3.6) and using *AS2* it immediately follows that:

$$\sum_{(i_1,i_2)\in\mathcal{I}_{2,D}^{\mathrm{a}}} \varphi_q^{i_1 i_2}\, a_{i_1}^j a_{i_2}^j = 0 \quad \forall\, q \in \mathcal{Q},\ \ \forall\, 1 \le j \le S\,. \tag{3.10}$$

This system of equations describes the relation between the unknown coefficients of the matrix $\mathbf{A}$ and the *known* set of coefficients in $\mathbf{\Phi}$. Letting $\mathbf{z} = [z^1, \ldots, z^D]^T$ and defining the $D$-variate polynomial functions:

$$f_q(\mathbf{z}) \triangleq \sum_{(i_1,i_2)\in\mathcal{I}_{2,D}^{\mathrm{a}}} \varphi_q^{i_1 i_2}\, z_{i_1} z_{i_2} \quad \forall\, q \in \mathcal{Q}\,, \tag{3.11}$$

system (3.10) can be written as:

$$f_q(\mathbf{a}_j) = 0 \quad \forall\, q \in \mathcal{Q}, \quad \forall\, 1 \le j \le S\,. \tag{3.12}$$

Hence, all columns $\mathbf{a}_j$ of $\mathbf{A}$ satisfy the system $\{f_q(\mathbf{z}) = 0\}_{q\in\mathcal{Q}}$. Consequently, if we can find the solutions of this system (which is known because the coefficients in $\mathbf{\Phi}$ are known), we can recover $\mathbf{A}$ from the sensor correlation functions only.
Equation (3.11) reveals that all terms in each function $f_q(\mathbf{z})$ have degree two. This implies that $f_q(\mathbf{z})$ is *homogeneous of degree two*, also noted as 2-homogeneous, meaning that:

$$f_q(\eta\mathbf{z}) = (\eta)^2 f_q(\mathbf{z}) \qquad \forall\, q \in \mathcal{Q}, \quad \eta \in \mathbb{R} \quad \mathbf{z} \in \mathbb{R}_D\,.$$

From this property, it directly follows that:

$$f_q(\mathbf{v}) = 0 \quad \forall\, q \in \mathcal{Q} \implies f_q(\eta\mathbf{v}) = 0 \quad \forall\, q \in \mathcal{Q},\ \eta \in \mathbb{R}\,.$$

Hence, if $\mathbf{v}$ is a solution of the system $\{f_q(\mathbf{z}) = 0\}_{q\in\mathcal{Q}}$, then also $\eta\mathbf{v}$ is a (an equivalent) solution for all $\eta \in \mathbb{R}$. This is a logical result of the so-called *scaling indeterminacy* inherent to MIBI [6]. Because of this, we are allowed to impose a norm constraint, say $\|\mathbf{z}\| = 1$ on the solutions of the system. Since the set of values of $\mathbf{z}$ for which $f(\mathbf{z}) = 0$ defines the zero contour level of a function $f(\mathbf{z})$, solving the system $\{f_q(\mathbf{z}) = 0\}_{q\in\mathcal{Q}}$ boils down to finding the intersections between the zero contour levels of the functions $f_1(\mathbf{z}), \ldots, f_Q(\mathbf{z})$. The homogeneity property of the polynomials in our system implies that these zero contour levels are cones in the $D$-dimensional Euclidian space. Hence, geometrically, solving $\{f_q(\mathbf{z}) = 0\}_{q\in\mathcal{Q}}$ means finding the intersections between $Q$ cones.

## 4. HOMOTOPY METHOD

Homotopy methods provide a deterministic means for solving a system of nonlinear equations by smoothly deforming the known solutions of a simple start system into the desired solutions of the target system [5]. They are based on the so-called *path following* techniques. In this section, we give a brief overview of the basic principles of homotopy methods. Excellent discussions can be found in several articles and books [4, 5]. The target system, i.e. the system to be solved, is denoted by $\mathbf{p}(\mathbf{z}) = \mathbf{0}$, and the start system by $\mathbf{g}(\mathbf{z}) = \mathbf{0}$. Homotopy methods operate in two stages. Firstly, the (expected) number of solutions and the structure of the target system are exploited to construct a start system. This start system is embedded in the (convex) *homotopy* defined as:

$$\mathbf{h}(\mathbf{z}, \lambda) = \gamma(1-\lambda)\,\mathbf{g}(\mathbf{z}) + \lambda\,\mathbf{p}(\mathbf{z}) \qquad \forall\,\lambda \in [0, 1], \tag{4.1}$$

where $\lambda \in \mathbb{R}$ is the so-called *continuation parameter* and $\gamma$ is a randomly chosen fixed constant [5]. Secondly, as $\lambda$ is increased from 0 to 1, numerical path following or *continuation* methods, trace the paths $\mathbf{z}(\lambda)$ defined implicitly by $\mathbf{h}(\mathbf{z}(\lambda), \lambda) = \mathbf{0}$ from the solutions of the start system to the solutions of the target system. In theory, each path converges to a geometrically isolated solution [5]. Because polynomial systems are very suited for homotopy methods [4], we can use a very simple path following method which proceeds as follows.
Suppose that $\mathbf{z}(\lambda)$ is a solution of $\mathbf{h}(\mathbf{z}(\lambda), \lambda) = \mathbf{0}$ for a certain (known) value of $\lambda$. Then, this solution is an approximate solution of the slightly deformed system $\mathbf{h}(\mathbf{z}(\lambda{+}\Delta\lambda), \lambda{+}\Delta\lambda) = \mathbf{0}$, where $\Delta\lambda$ is a small increment. The refined solution of the deformed system can be found by first *predicting* $\mathbf{z}(\lambda + \Delta\lambda)$ by means of e.g. an Euler step, and then *correcting* the prediction by means of any local zero finding method, e.g. Newton's method [5]. Proceeding this way from the initially known solutions at $\lambda = 0$ to the initially unknown solutions at $\lambda = 1$, we finally end up at the solutions of the target system. This is the basic rationale of homotopy methods. Although many much more sophisticated and optimized methods exist, for our demonstration purposes the above described method suffices.
Now we will apply the ideas outlined above to the system of polynomial equations derived in Section 3 for an example of a specific MIBI problem with $D = 3$ and $S = 4$, the details of which are presented in Section 5. For that example, it can be shown (see Section 5) that the number $Q$ of independent equations is 2. Let the equations of target system $\mathbf{p}(\mathbf{z})$ in (4.1) be given by $p_1(z_1, z_2, z_3) \triangleq f_1(z_1, z_2, z_3)$ and $p_2(z_1, z_2, z_3) \triangleq f_2(z_1, z_2, z_3)$ respectively. We choose the following start system:

$$\begin{cases} g_1(z_1, z_2, z_3) \triangleq (z_1)^2 - (\beta_1)^2 (z_3)^2 = 0 \\ g_2(z_1, z_2, z_3) \triangleq (z_2)^2 - (\beta_2)^2 (z_3)^2 = 0 \end{cases}, \tag{4.2}$$

where $\beta_1$ and $\beta_2$ are randomly chosen fixed complex constants [4, 5]. This system has the following solutions for $\mathbf{z}$:

$$\mathbf{z} \triangleq \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ z_3 \end{bmatrix}, \begin{bmatrix} \beta_1 \\ -\beta_2 \\ z_3 \end{bmatrix}, \begin{bmatrix} -\beta_1 \\ \beta_2 \\ z_3 \end{bmatrix}, \text{ and } \begin{bmatrix} -\beta_1 \\ -\beta_2 \\ z_3 \end{bmatrix}. \tag{4.3}$$

As we have explained in Section 3, we may impose the norm constraint $\|\mathbf{z}\| = 1$. In fact, this means including the equation $(z_1)^2 + (z_2)^2 + (z_3)^2 = 1$ to homotopy system (4.1). In total, we obtain the following homotopy for $\lambda \in [0, 1]$:

$$\begin{bmatrix} h_1(z_1, z_2, z_3, \lambda) \\ h_2(z_1, z_2, z_3, \lambda) \\ h_3(z_1, z_2, z_3, \lambda) \end{bmatrix} = \begin{bmatrix} \gamma(1-\lambda) g_1(z_1, z_2, z_3) + \lambda f_1(z_1, z_2, z_3) \\ \gamma(1-\lambda) g_2(z_1, z_2, z_3) + \lambda f_2(z_1, z_2, z_3) \\ (z_1)^2 + (z_2)^2 + (z_3)^2 - 1 \end{bmatrix}. \tag{4.4}$$

**Algorithm 1** MIBI based on SOS and homotopy ($D=3, S=4$).

1: *Compute/estimate sensor correlation matrix* $\mathbf{C}^x$ *in* (3.8)*;*

2: *Compute* $\boldsymbol{\Phi}$ *in* (3.7) *from the SVD of* $\mathbf{C}^x$*; now the system* $\{f_q(\mathbf{z})=0\}_{q\in\mathcal{Q}}$ *to be solved is known (see* (3.11)*);*

3: *Define a fine grid* $\mathcal{G}$ *on* $\lambda\in[0,1]$*, e.g.* $\mathcal{G}\triangleq\{0,\Delta\lambda,2\Delta\lambda,\ldots,1\}$*;*

4: *For each solution* $\mathbf{z}(0)$ *of the start system (see* (4.3)*), and each subsequent* $\lambda\in\mathcal{G}\setminus\{0\}$*, do:*
- *Use one step of Eulers method to predict* $\mathbf{z}(\lambda)$ *from* $\mathbf{z}(\lambda-\Delta\lambda)$ *as follows:* $\mathbf{z}:=\mathbf{z}+\frac{d\mathbf{z}}{d\lambda}\cdot\Delta\lambda$*;*
- *Use several steps of Newton's method to correct* $\mathbf{z}(\lambda)$*, i.e. find the solution of* $\mathbf{h}(\mathbf{z}(\lambda),\lambda)=\mathbf{0}$ *by iterating:*

$$\mathbf{z}:=\mathbf{z}-\left[\nabla_{\mathbf{z}}\mathbf{h}(\mathbf{z},\lambda)\right]^{-1}\mathbf{h}(\mathbf{z},\lambda).$$

A possible homotopy algorithm for finding all unique solutions of the system $f_1(\mathbf{z})=f_2(\mathbf{z})=0$ is given in Alg. 1. The gradient $\frac{d\mathbf{z}}{d\lambda}$ in step 4 can be found from the equation obtained by taking the derivative of $\mathbf{h}(\mathbf{z}(\lambda),\lambda)=\mathbf{0}$ w.r.t. $\lambda$. Furthermore, $\nabla_{\mathbf{z}}\mathbf{h}(\mathbf{z},\lambda)$ denotes the Jacobian matrix of $\mathbf{h}(\mathbf{z},\lambda)$ w.r.t. $\mathbf{z}$.

## 5. EXAMPLE

In this section, we will apply the theory developed above to an example of a MIBI scenario with $D=3, S=4$. Let $\mathbf{A}$ be given by:

$$\mathbf{A}=\begin{bmatrix}-0.6804 & 0.5774 & -0.2673 & 0.5538\\ 0.2722 & 0.5774 & -0.8018 & -0.8308\\ 0.6804 & 0.5774 & 0.5345 & -0.0554\end{bmatrix}.$$

We have used 30000 samples of artificial source signals generated according to the following AR(2) model:

$$s_j[n]=c_j^1 s_k[n-1]+c_j^2 s_j[n-2]+w_j[n]\,, \qquad (5.1)$$

where $w_k[n]$ is a stationary white Gaussian noise sequence, and the pair of coefficients $\left(c_j^1,c_j^2\right)$ determines the pole positions of the associated AR(2) all-pole filter. Thus, the source signals possess an 'AR(2) temporal structure'. In addition, the source signals are also normalized to have unit variance. Since the data is stationary, only the nonwhiteness can be exploited. We choose the ROS (see Section 2) to be the set of lags $\mathcal{T}\triangleq\{1,\ldots,7\}$ (due to stationarity, absolute times are irrelevant and omitted). In Fig. 2 the chosen pole pairs $\left(c_j^1,c_j^2\right)$ and the corresponding source correlation vectors (functions) $\tilde{\mathbf{r}}_{jj}^s$ on the lag interval $\{-7,\ldots,0,\ldots,7\}$ are depicted for $1\le j\le S$.

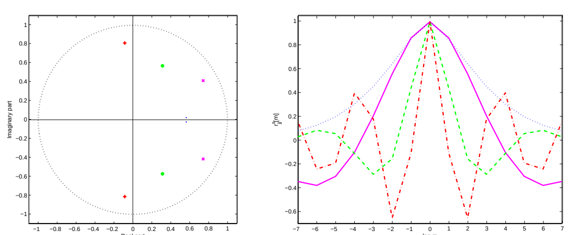

Figure 2: Source pole pairs and correlation functions.

The sensor signals $\mathbf{x}[n]$ are obtained by mixing the source signals $\mathbf{s}[n]$ with $\mathbf{A}$ and adding noise $\boldsymbol{\nu}[n]$ according to model (2.1). The standard deviations of the noise-free parts of the sensor signals $x_1[n]$, $x_2[n]$, and $x_3[n]$ are given by 1.09, 1.32 and 1.04 respectively. The noise signals $\nu_1[n]$, $\nu_2[n]$, and $\nu_3[n]$ are mutually statistically independent white Gaussian noise sequences with standard deviation 0.5. From the sensor signals, we estimate the sensor correlation functions $\{r_{i_1 i_2}^x[m]\}_{1\le i_1,i_2\le 3}$ for all lags $m\in\mathcal{T}$ by averaging products of the form $x_{i_1}[n]x_{i_2}[n-m]$ over all 30000 time samples $n$. These functions form the input for Alg. 1. In order to apply this algorithm, we first have to determine $Q$, for which we need to know the dimension $d^x$ of the linear space spanned by the sensor correlation row vectors. This dimension depends on the mixing matrix $\mathbf{A}$ and the dimension $d^s$ of the linear space spanned by the source auto-correlation row vectors. In general, it can be shown that $d^x=d^s$ with probability one if $\mathbf{A}$ is drawn from a continuous probability distribution. In our current example, $d^x=d^s=4$. Hence, there are $Q=\frac{1}{2}D(D+1)-d^x=2$ independent polynomial equations $f_1(z_1,z_2,z_3)=0$ and $f_2(z_1,z_2,z_3)=0$ in the system to be solved. The coefficients of the two polynomials are obtained from the two left singular vectors that correspond to smallest singular values of $\mathbf{C}^x$. More specifically, suppose that the 'ordered' SVD of $\mathbf{C}^x$ is given in standard form by $\mathbf{C}^x=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^T$. Then, the row $\tilde{\boldsymbol{\varphi}}_1$ containing the coefficients of $f_1(z_1,z_2,z_3)$ is the transpose of the $(d^x+1)$-th (5-th) column of $\mathbf{U}$, and similarly the row $\tilde{\boldsymbol{\varphi}}_2$ containing the coefficients of $f_2(z_1,z_2,z_3)$ is the transpose of the $\left|\mathcal{K}_D^x\right|$-th (6-th) column of $\mathbf{U}$. Now running Alg. 1 yields the following estimate of $\mathbf{A}$:

$$\hat{\mathbf{A}}=\begin{bmatrix}-0.6830 & 0.2648 & 0.5752 & 0.5502\\ 0.2935 & 0.8077 & 0.5794 & -0.8337\\ 0.6688 & -0.5268 & 0.5774 & -0.0467\end{bmatrix}.$$

The reader can easily verify that, apart from the indeterminacies mentioned in Section 2, this is close a estimate of $\mathbf{A}$.

## 6. CONCLUSIONS

We have presented a batch-mode MIBI method that is based on SOS and a homotopy method. As an example, an algorithm specific for the scenario with 3 sensors and 4 sources was presented, thereby demonstrating the ability to estimate more sources than sensors, and the applicability of homotopy methods. The method can easily be generalized to higher order statistics. A topic for future research is to make the algorithm adaptive.

## 7. REFERENCES


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