A NEW NOISE MODEL FOR DESIGNING SUPERDIRECTIVE BEAMFORMERS FOR SPECIAL APPLICATIONS

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ABSTRACT

The design of superdirective beamformers is based on the minimum variance distortionless response solution (MVDR) for a spherically or cylindrically isotropic noise field. In this contribution a new noise model is given. It excludes parts of the noise field near the pre-described look direction from the isotropic assumption. This leads to a new but suboptimal beamformer when measuring the directivity index, but it can provide a better robustness against steering errors. Furthermore, in some applications like recording orchestras the desired sources certainly are in front of the array and the audience as the disturbances are on the back side. Therefore, assuming an isotropic noise field is not the optimal choice.

1. INTRODUCTION

It is well-known that the frequency-domain MVDR solution for non-adaptive microphone arrays can be reduced to just one equation [1]. Under the assumption of a homogeneous noise field we get[1]:

$$\boldsymbol{W} = \frac{\boldsymbol{\Gamma_{XX}}^{-1}\boldsymbol{d}}{\boldsymbol{d}^H\boldsymbol{\Gamma_{XX}}^{-1}\boldsymbol{d}}, \tag{1}$$

where

$$\boldsymbol{\Gamma_{XX}} = \begin{pmatrix} 1 & \Gamma_{X_0X_1} & \cdots & \Gamma_{X_0X_{N-1}} \\ \Gamma_{X_1X_0} & 1 & \cdots & \Gamma_{X_1X_{N-1}} \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma_{X_{N-1}X_0} & \Gamma_{X_{N-1}X_1} & \cdots & 1 \end{pmatrix} \tag{2}$$

is a coherence matrix. Each element consists of the complex coherence defined as

$$\Gamma_{X_iX_k} = \frac{\Phi_{X_iX_k}}{\sqrt{\Phi_{X_iX_i}\Phi_{X_kX_k}}}, \tag{3}$$

where $\Phi_{X_iX_k}$ is the power spectral density (PSD) between sensor i and k. Furthermore, $\boldsymbol{d}$ is the propagation vector of the desired signal.

[1]The frequency variable is omitted.

Many traditional designs, including the delay-and-sum beamformer and the superdirective beamformer, are based on (1) [2]. However, these designs are not the optimal choices for all possible applications. In the following section we present a new noise model, which excludes some regions of the isotropic noise fields in order to get a flexible design for special application problems. Some of these applications will be discussed and design examples will be given.

2. NOISE MODEL DESCRIPTION

In order to describe isotropic noise fields, we assume that infinite noise sources are randomly placed on a circle or a sphere with an infinite radius. Additionally, each noise source has a white PSD (not necessarily) and a random phase.
The coherence between two sensors for the two-dimensional case is given by:

$$\begin{aligned} \Gamma(\omega) &= \frac{1}{2\pi}\int_0^{2\pi} \exp\left(j\frac{\omega d\cos(\theta)}{c}\right)d\theta \qquad (4) \\ &= J_0\left(\frac{\omega d\cos(\theta)}{c}\right), \qquad (5) \end{aligned}$$

where d denotes the distance between the microphones, c the speed of sound, and J_0 the zeroth-order Bessel-function of the first kind. This design cannot be used directly, as the white noise gain is very high at low frequencies, but a constrained design can easily be computed by using an additional parameter μ [2].

$$\Gamma(\omega) = \frac{J_0\left(\frac{\omega d\cos(\theta)}{c}\right)}{1+\mu}. \tag{6}$$

This constraining procedure is assumed for the rest of the paper.
This model can be extended by excluding some regions near the look direction θ_0 defined as the angle between the sensor axis and the position of the desired source. The excluded sector can be described by an additional

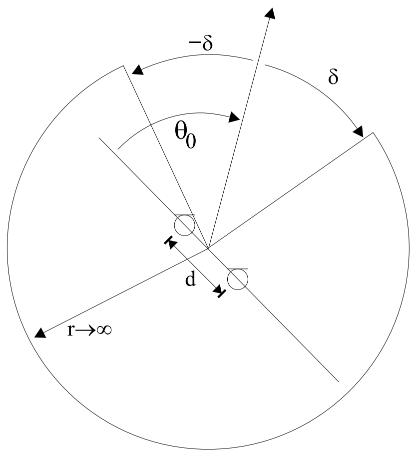

Figure 1: A two-dimensional noise model with one sector of $\pm\delta^\circ$ specified as non-radiating noise.

parameter δ (see figure 1 for a detailed model).

$$\Gamma(\omega, \theta_0, \delta) = \frac{1}{2(\pi - \delta)} \int_{\theta_0+\delta}^{\theta_0-\delta+2\pi} \exp\left(j\frac{\omega d \cos(\theta)}{c}\right) d\theta \tag{7}$$

Unfortunately, the solution of the integral can only be approximated by numerical integration. One efficient algorithm for this is the Romberg-method [3].

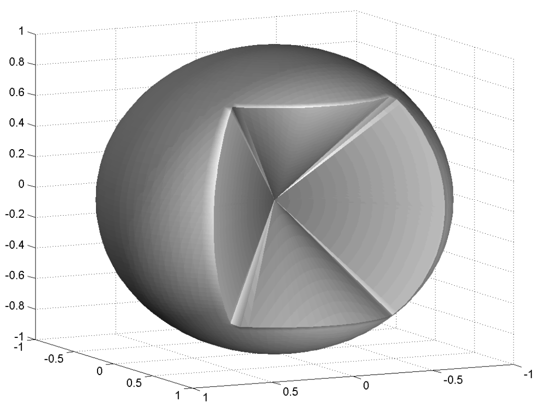

Figure 2: A three-dimensional noise model with one sector of $\pm\pi/4$ specified as non-radiating noise.

Of course, this approach can be extended to three dimensions. The coherence in this case is given by

$$\Gamma(\omega, \theta_0, \phi_0, \delta, \rho) = \frac{1}{4(\pi - \delta)(\pi - \rho)} \cdots \tag{8}$$
$$\int_{\theta_0+\delta}^{\theta_0-\delta+2\pi} \int_{\phi_0+\rho}^{\phi_0-\rho+2\pi} e^{\frac{j\omega d \cos(\theta)}{c}} \sin(\phi) d\phi d\theta,$$

where ρ denotes the excluding angle in the azimuth plane. An example of the noise model with a sector of $\pm\pi/4$ for the elevation and the azimuth angle (θ, ϕ) is shown in figure 2. The computational costs for the numerical solution are high, but we have to compute the coefficients of the array only once.

3. DESIGN EXAMPLES

In the following theoretical study we are using the beam-pattern, the directivity index (DI), and the front-to-back ratio (F) measures [4, 5]. We examine line arrays consisting of 5 equi-spaced omnidirectional microphones. The sensor spacing is set to 4 cm, to avoid spatial aliasing. The array is steered to the endfire direction ($\theta_0 = \pi$) and the white noise constraint is set to $\mu = 0.001$. For a detailed description of the constraining procedure see [6, 2].
For comparison figure 3 shows the beam-pattern for the isotropic design ($\delta = 0$). In the following figures δ is increased by $\pi/4$ for each figure (see fig. 4-6). Comparing these figures, leads to the following results:

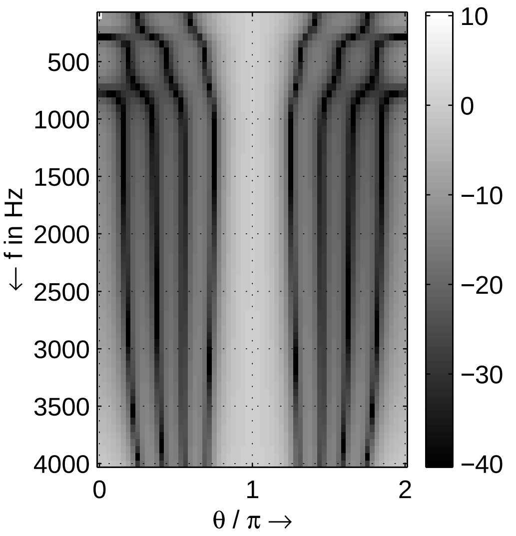

Figure 3: Beam-pattern of a superdirectional beamformer, designed for an isotropic noise field.

- Increasing δ to $\pi/4$ has only little impact on the main-lobe of the beam-pattern, but it increases the reduction of the side-lobes significantly.

- By using $\delta = \pi/2$ the main-lobe has been broadened, but noise sources coming from the back side will be suppressed almost completely.

- Setting $\delta = 3\pi/4$ extends the main-lobe further. Additionally, an amplification at higher frequencies occurs, caused by the spatial aliasing problem. Optimizing the noise-filed near the spatial aliasing regions will lead to amplifications in the 'don't care' regions. This holds also for $\delta = \pi/2$.

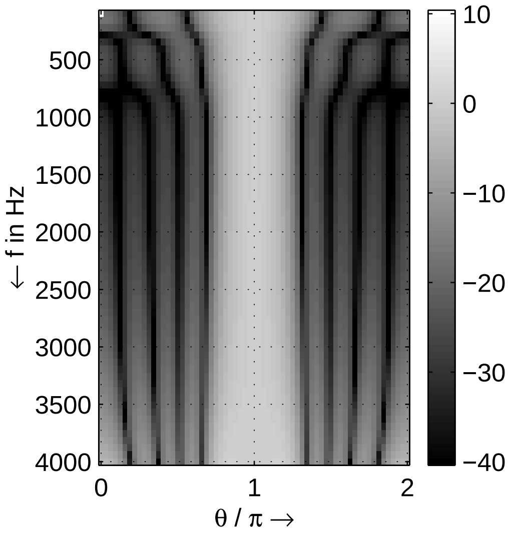

Figure 4: Beam-pattern of a superdirectional beamformer, designed with eq. 7 and $\delta = \pi/4$.

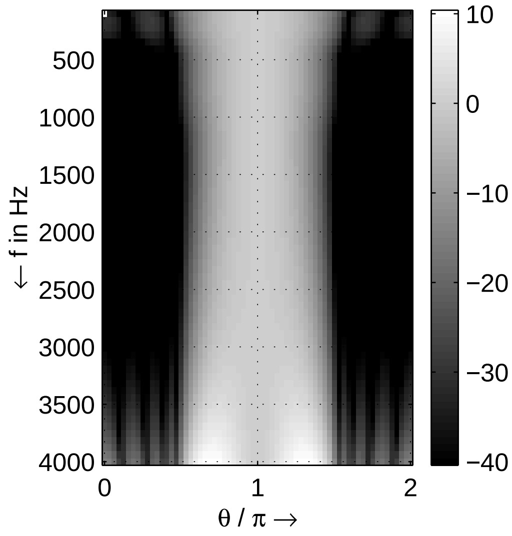

Figure 5: Beam-pattern of a superdirectional beamformer, designed with eq. 7 and $\delta = \pi/2$.

Additionally, figure 7 depicts the directivity index for all four designs. Obviously, the isotropic design is the best choice for optimizing the DI. The negative DI at high frequencies for $\delta \geq \pi/2$ is caused by the amplification near the look-direction.
However, when analyzing the front-to-back ratio the results differ significantly. Setting $\delta = \pi/2$ leads to a very high front-to-back ratio, whereas the isotropic design is not the optimal choice in this case.
A good compromise can be reached by using $\delta = \pi/4$. We are only losing 1 dB in the DI, but we are getting $\approx$ 10 dB in the front-to-back measure.

4. APPLICATIONS

However, why should we use this new scheme? The following list shows some possible extensions to known applications for non-adaptive beamformers:

- Hearing aids: Non-adaptive arrays were tested successfully to improve the speech quality and the spatial impression for hearing-impaired people [7]. Doerbecker has shown that the suppression of noise sources from the back side will further improve the ability to follow one speaker's speech in a group discussion [8]. With $\delta = \pi/2$ and (7) a very good suppression can be obtained.

- Recording Orchestras: All desired sources are in front and all unwanted disturbances are in the back of the array. Therefore, the front-to-back ratio has to be optimized. Optimizing this measure in order to design gradient microphones leads to supercardioid microphones, which can be used for recording instruments [4, 5]. By using (8) special arrays for this purpose can be designed.

- Tele-Conferencing: In most cases the speaker or the group of speakers are in front of the array system. Furthermore, the height between people does not alter much. Therefore, an array can be designed with a small elevation sector and a large azimuth sector for optimal recording conditions.

5. CONCLUSION

In this paper we have shown a class of beamformers designed by using modified noise fields. The modifications were computed by excluding regions near the look-direction from the isotropic assumption typically used by beamformer design rules. This leads to designs with very high front-to-back ratios, but the results for the directivity index are worse in comparison to the superdirectional beamformer design. The additional degree of freedom for the design procedure can be used to improve the behaviour of beamformers, especially for some applications like hearing aids.

6. REFERENCES

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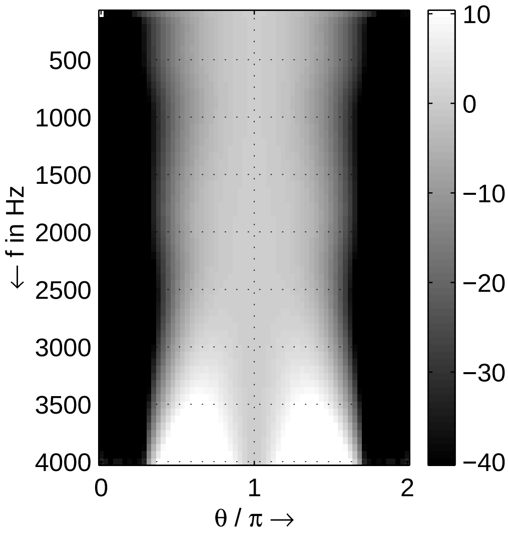

Figure 6: Beam-pattern of a superdirectional beamformer, designed with eq. 7 and $\delta = 3\pi/4$.

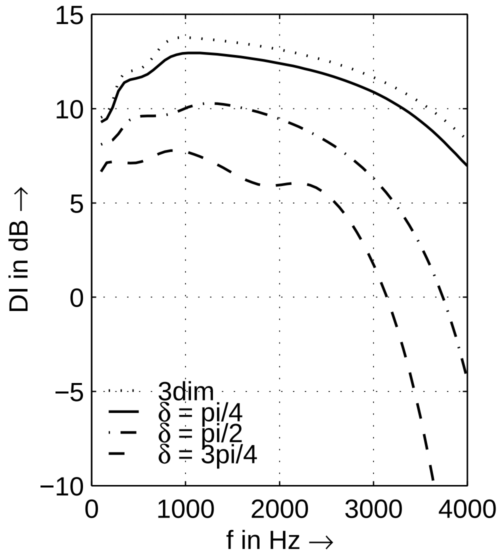

Figure 7: Directivity index for different designs

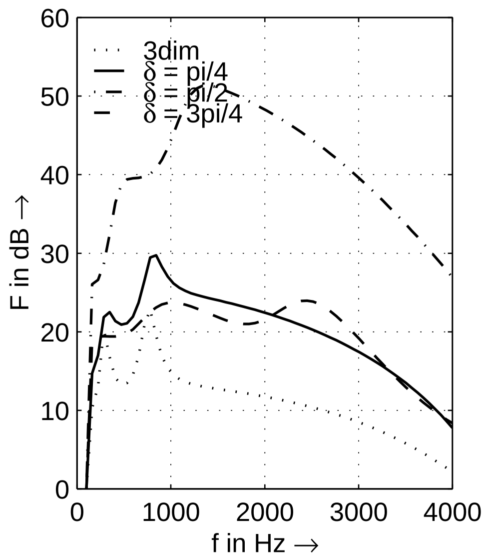

Figure 8: Front to back ratio for different designs

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