IDENTIFICATION OF A NONLINEAR EEG GENERATING MODEL

Ola Markusson and Torsten Bohlin

S3- Automatic Control
Royal Institute of Technology (KTH)
S-100 44 Stockholm, Sweden
`olam@s3.kth.se bohlin@s3.kth.se`

ABSTRACT

This contribution is a study of a method for identification of nonlinear stochastic models. Models generating electroencephalograms (EEG), based on neurophysiological knowledge are studied, [1]. A model-based analysis of single evoked potentials is also suggested. The main idea behind the identification is to use an inverted model, since no general predictor is available for nonlinear models. A maximum likelihood (ML) method is used to estimate the structure and the parameters of the model. To utilize *a priori* knowledge a 'grey-box' approach is taken.

1. THE EEG GENERATING MODEL

A neurophysiologically-based model, developed by Jansen and Rit [1], is the basis for this project. The model of a cortical column is simulating electrical brain activity. A set of nonlinear differential equations describes the model as

$$\dot{x}(t) = f(x(t), \theta) + g(\theta)w(t) \quad (1)$$

$$y(t) = h(x(t), \theta), \quad (2)$$

where $x(t)$ is a state vector, θ is the parameter vector to be estimated and $y(t)$ denotes the output. The model input is represented by $w(t)$ and is characterized by a probability density function ϕ_w. In this case we consider a zero mean gaussian process with unit variance. The parameters, mean m and standard deviation σ are included in the parameter vector and need to be estimated.

The cortical column is modeled by a population of main cells interacting via feedback branches, as shown in Figure 1. Each branch is either direct or represented by a transformation. This transformation, of an average pulse density into an average potential, may be either excitatory $h_e(t)$ or inhibitory $h_i(t)$. The excitatory transfer function, e.g., is described by

$$h_e(t) = 0, \; t < 0 \quad (3)$$

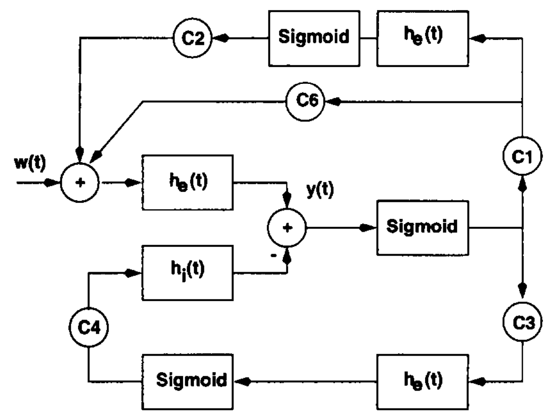

Figure 1: Detailed block diagram of the SISO system.

$$h_e(t) = Aate^{-at}, \; t \geq 0 \quad (4)$$

where A is included in θ and a is considered to be known. The direct connections are only affected by a connectivity gain C_i. A sigmoid function described by

$$S[x_i(t)] = 2e_0/(1 + e^{r(v_o - x_i(t))}), \quad (5)$$

causes the nonlinear behavior of the model. Only some of the parameters in (5) are estimated while others are treated as known. The output $y(t)$ is the difference between the outputs of the excitatory and the inhibitory transfer functions.

Both single input, single output (SISO) models and multi-input, multi-output (MIMO) models are tested. The tested MIMO model, shown in Figure 2, consists of two coupled SISO models S_i. The SISO models are connected by an excitatory transfer function and a connectivity gain. The MIMO model with two inputs and two outputs is also used for exploring flash visual evoked potentials (FVEPs). To simulate a FVEP a pulse-like input component (PLIC), described by

$$P(t) = 0, \; t < t_0 \quad (6)$$

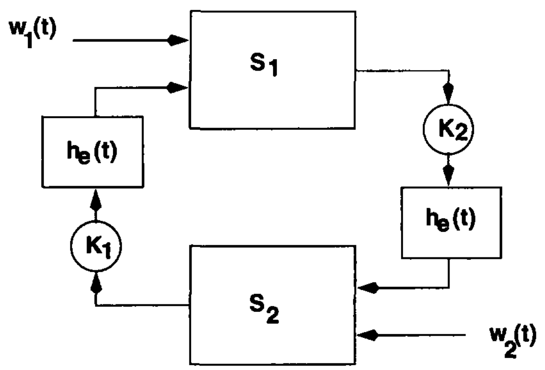

Figure 2: Block diagram of the MIMO system. S_i is a SISO system described in Figure 1.

$$P(t) = q(\frac{t-t_0}{\eta})^n e^{-\frac{t-t_0}{\eta}}, \quad t \geq t_0 \tag{7}$$

represents the impulse density related to the visual input, [1]. Only the parameter q, representing the amplitude, is estimated. The PLIC is added to one of the inputs of the MIMO model.

2. IDENTIFICATION

'Grey-box' identification utilizes *a priori* information along with experimental data. The basic ideas for interactive design and identification of a stochastic model are suggested in [2]. The procedure is based on sequences of testing of gradually refined model equations. Hence, the procedure starts with a simple model which is later expanded. In this study the following procedure is used, [3], to achieve the simplest possible model structure, not contradicted by data.

1. *Initialization:* A first 'root' model is constructed. It is supposed to be as simple as possible. Initial parameter values are also set.
2. Repeat while the model is falsified:
   (a) Specify a tentative free parameter set and fit the free parameters. If no more free parameters are available, then expand the set of equations.
   (b) Test the model against a set of alternative models.
   (c) Evaluate the test statistics.
   (d) Appraise the model, if any of the alternatives is better, then falsify the model. Go back to 2.(a) starting with the best of the alternative models.

The above described procedure is not fully automated and therefore needs a certain interaction by the user. Such interaction involves the selection of alternative models and how the initial parameter values are chosen.

The testing, under point 2.(b), is based on the likelihood of the model, parameterized by θ, and that of the alternative model, parameterized by θ'. Here, the likelihood of the parameter vector θ is written as

$$L(\theta \mid y) = \phi_y(y \mid \theta), \tag{8}$$

where ϕ_y is the conditional probability density function (PDF) of y given the parameter vector θ. As shown in [4], ϕ_y can be computed as

$$\phi_y(y \mid \theta) = \phi_w(W(y)) \left| \frac{d[W(y)]}{dy} \right|. \tag{9}$$

where ϕ_w is the PDF of w. In (9), W denotes the inverse of the relation between the input and the output. The minimization of the negative log likelihood function

$$Q(\theta) = -\log L(\theta|y) \tag{10}$$

is performed for each iteration by a Levenberg Marquardt algorithm.

The falsification test, 2.(c), is based on the likelihood ratio. The confidence in a reject decision of the model represented by θ vs. an alternative model represented by θ' is

$$\gamma = \chi^2_r[2Q(\hat{\theta}) - 2Q(\hat{\theta}')]. \tag{11}$$

In (10), $\hat{\theta}$ and $\hat{\theta}'$ minimizes the loss functions $Q(\hat{\theta})$ and $Q(\hat{\theta}')$ respectively. If $\gamma > (1 -$ *the risk level*), then the model parameterized by θ will be rejected. In (11), r is the number of degrees of freedom, $r = dim(\theta') - dim(\theta)$.

3. INVERTIBILITY

The criteria for invertibility are important and they can be summarized in two points,

- The mapping from y to w has to be one-to-one and onto. Hence, the number of noise inputs to the model has to be the same as the number of measured output signals.
- The mappings have to be differentiable.

In our case, with the model written as in (1), the criteria for invertibility stated in, e.g., [5] can be used. If the system is linearized at a point x_0 then the system equation can be written as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bw(t) \\ y(t) &= Cx(t) \end{aligned} \tag{12}$$

where A is the linearization of $f(x(t), \theta)$ computed as

$$A = \frac{\partial f(x)}{\partial x}|_{x=x_0}. \quad (13)$$

The system is invertible in the point, $x = x_0$, if

$$V^* \cap Im(B) = 0 \quad (14)$$

where V^* is the largest subspace of ker(C) satisfying

$$AV^* \subset V^* + Im(B). \quad (15)$$

The above criteria is a local test around the point x_0. The test of invertibility can therefore be applied as a function which tests the invertibility "on line".

4. CONVERGENCE

We will only briefly mention the main results of [6], where the convergence of the ML estimator for an invertible stochastic model was investigated. According to Ljung [7], the identification procedure is based on three entities; the data record, the set of models and the criterion.

Ljung lists three conditions on the data where we will especially note the condition of *exponential forgetting*. This condition means that the remote past of the process is forgotten at an exponential rate. Hence, "good" approximations of $y(t)$ can be made that are independent of the past.

The inverted model is similar to the probabilistic model defined by Ljung [7] (p. 185), although, here, $w(t)$ is computed by the inversed model instead of a prediction model. According to Ljung, the model has to be restricted in two ways. First, the rate at which the inverse transformation W may increase with y has to be restricted. Also, the model and its derivatives with respect to θ have to be exponentially stable. It should be noted that these conditions concern the model, not the data, and it is *our* model.

It is shown in [6] that the ML estimator will converge to a parameter set where the likelihood function is maximal. Also, for the special case when the model set contains the "true model" the estimator will converge to a set containing the true model.

5. ANALYSIS OF SINGLE EVOKED POTENTIALS

We suggest a model based method for analyzing single FVEP. First, the model parameters are estimated on both pre- and poststimulus data, including the parameters describing the properties of the PLIC. Thereafter, the model is simulated both with and without

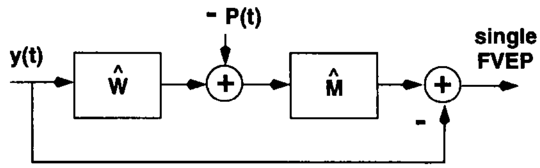

Figure 3: *Schematic for the estimation of a single FVEP.*

the PLIC and the resulting outputs are compared. In Figure 3, a block diagram is shown where the measured signal, $y(t)$, is used to estimate the parameters of the inversed model , $\hat{W}$, and the PLIC, $P(t)$. Thereafter, $P(t)$ is subtracted from the noise sequence and the estimated model $\hat{M}$ is simulated using the 'new' noise sequence. The single FVEP is computed as the difference between the two signals. Here, we assume that the ongoing activity is unaffected by the evoked potential (EP). This assumption combined with the assumption that the EP remains the same from trial to trial are used by traditional methods. Methods, analysing EPs, based on averaging a large number of trials. Although, our assumption is somewhat contradicted by the use of a nonlinear model, it will make comparisons with traditional methods possible.

6. IDENTIFICATION RESULTS

The identification algorithm is tested on sequences of two seconds of simulated data. The sampling frequency is 500 Hz, hence, there is 1000 samples in each sequence. The parameter vector used for the simulation will be denoted θ_0 and the estimated vector $\hat{\theta}$. Both a SISO model and a MIMO model are tested. The latter also under generation of flash visual evoked potentials (FVEPs). In the case of FVEP the amplitude of the pulse-like component, is estimated.

We will start by showing the detailed results of the identification procedure for a SISO model. Since the procedure is similar for the more complex MIMO model, only the main parts of the results will be presented. Finally, the results from the estimation of the FVEP amplitude will be shown along with the usage of the analysis scheme suggested in Section 5.

The risk level, described in Section 2, is set to 5%. Hence, if γ is larger than 95 % the model will be rejected.

A SISO case

As pointed out in Section 2, a 'root' model has to be specified by the user to begin with. The root model

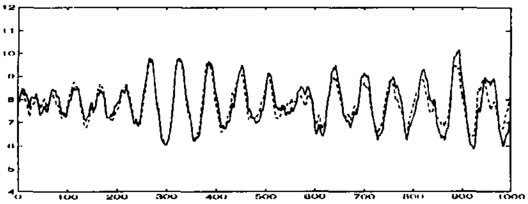

Figure 4: Validation for the SISO model. The solid line corresponds to θ_0 and the dashed line to $\hat{\theta}$.

in our case only consisted of the noise model and the first filter $h_e(t)$. To begin with, three parameters were freed, the parameters representing the mean and the variance of the noise and the parameter A, characterizing the gain of $h_e(t)$, see (3). The results from the parameter estimation is shown in Table 1 as trial 1. All the C parameters were kept equal to zero and the resulting negative logarithm of the likelihood, Q, is -2028. It should be noted that the value of Q is only valid for comparisons made on the same data sequence. To expand the model structure the inhibitory feedback branch was tested. Three more parameters were freed; C_3, C_4, and v_0 (included in the sigmoid function). Actually, there is another parameter, representing the gain of $h_i(t)$, but it is included C_4. When freeing the three parameters the value of Q decreased by 223 which is a significant reduction. Further expansion of the model structure involved the excitatory feedback parameterized by C_1 and C_2. As shown in Table 1, trial 3, the Q value dropped by 9 to -2260. Finally, the hypothesis of the direct feedback branch parameterized by C_6 was tested and found significant since the loss reduction was 81. The parameter vector used for simulating data is presented as θ_0 in Table 1.

Four sequences of data, simulated with θ_0 but with different noise realizations, were used to test the parameter variation. The parameter estimates of v_0, A, and σ were close to their actual values. However, the values of the connectivity parameters C_i and the mean of the noise, m, varied depending on the initialization. Different combinations of C_i and m gave almost the same value of Q. To validate these results, the model was simulated with another noise sequence using θ_0 and $\hat{\theta}$. An example of a simulation for validation using $\hat{\theta}$ from trial 4 in Table 1 is shown in Figure 4. This is a test that can only be made on simulated data, however, it suggests that the parameterization is non unique.

A MIMO case

The MIMO model consists of two SISO models, coupled by two excitatory branches as shown in Figure 2. The connectivity parameters K_1 and K_2 determines the degree of coupling. As long as K_1 and K_2 are kept equal to zero, the model can be considered as two separate SISO models.

An example of the estimation results are shown in Table 2. As shown, the parameter values differed considerably from their actual values in this case. However, in Figure 5, the results from a simulation using a new input realization is shown, and the result is considered to be satisfying.

Four different realizations were tested and the results were compared. The estimated values of the C parameters and m varied, and differed from their actual values. On the other hand, the simulations with a new input realizations gave satisfying results. We therefore conclude that there is a non unique parameterization of the model.

Table 1: *An example of the parameter estimates and the model expansion in a SISO case.*

Trial	v_0	A	m	σ	Q	ΔQ
1	-	3.32	240	59	-2028	-
2	6.4	5.8	263	29	-2251	223
3	5.9	3.1	289	40	-2260	9
4	5.8	2.7	291	53	-2341	81
θ_0	6.0	2.6	220	57		
Trial	C_1	C_2	C_3	C_4	C_6	
1	-	-	-	-	-	
2	-	-	20	13	-	
3	90	110	25	42	-	
4	114	82	27	36	24	
θ_0	125	100	31	32	20	

Table 2: *An example of the parameter estimates for a MIMO case.*

	v_{01}	A_1	m_1	σ_1	K_1
θ_{S1}	6.0	3.6	220	58	300
$\hat{\theta}_{S1}$	6.3	5.1	360	40	325
	C_{11}	C_{12}	C_{13}	C_{14}	C_{16}
θ_{S1}	108	86	27	27	10
$\hat{\theta}_{S1}$	47	17	20	25	4
	v_{02}	A_2	m_2	σ_2	K_2
θ_{S2}	5.0	2.6	220	58	400
$\hat{\theta}_{S2}$	5.3	2.2	341	67	412
	C_{21}	C_{22}	C_{23}	C_{24}	C_{26}
θ_{S2}	108	86	27	27	0
$\hat{\theta}_{S2}$	134	93	31	38	0

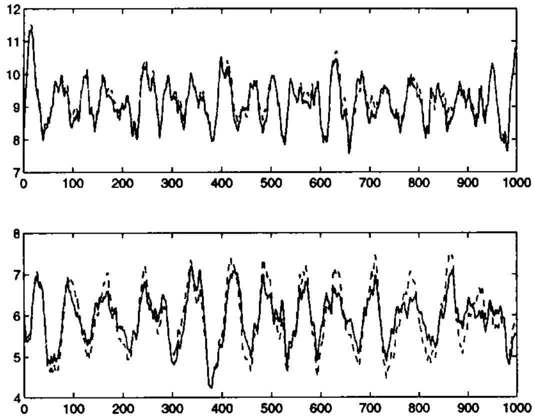

Figure 5: Validation for the MIMO model. The solid line corresponds to θ_0 and the dashed line to $\hat{\theta}$.

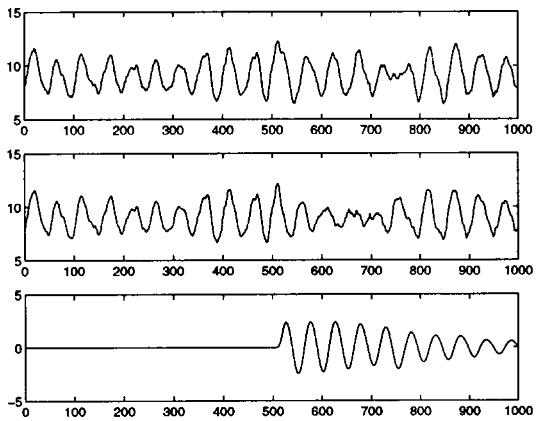

Figure 6: Estimated single FVEP. Top: the signal including the FVEP. Middle: FVEP excluded. Bottom: The difference between the top and the middle.

FVEP

The MIMO system estimated in the previous section was also stimulated by a PLIC, described in Section 1. The amplitude of the pulse, $q = 0.1$, was estimated and the scheme for estimating the single flash visual evoked potential (FVEP) was used. The amplitude parameter was freed last and the whole data sequence of data was used. In other words, both pre- and poststimulus data was used. The parameter estimates varied between 0.08 and 0.13, and was found significant in all cases.

In Figure 6, the results are shown from the estimation of a single FVEP. On top, is the simulated signal including the FVEP. Next is the simulated signal excluding the FVEP. At the bottom the difference and, hence, the suggested single FVEP is shown.

Summary

The correct model structure was estimated in all the tested cases. However, the estimated parameter values varied. Since the model shows different behavior for different parameter vectors it is hard to generalize the results to include all parameter combinations. The results should, therefore, be seen as examples.

In all the presented cases, the initial conditions (IC) have been treated as known. There are, however, other ways of treating the ICs. One way is to estimate them, hence add the ICs to the parameter vector. The disadvantage of doing that would be that the number of parameters to be estimated would almost double. Another way would be to use the property of exponential forgetting (see Section 4), by this assumption the first samples of the inverted signal should not be included in the computation of the likelihood function.

7. CONCLUSIONS

An identification procedure for nonlinear stochastic models, based on a priori information, is presented. The main idea behind the algorithm is to use an inverted model and an ML estimation method of the parameters. A case-study is performed on a nonlinear EEG generating model. The proposed method is only tested on simulated data so far. However, the results show that the method can be used for the validation of the ideas behind the models. Thus, the method may provide a tool for improved analysis of EEG.

8. REFERENCES

[1] B.H. Jansen and V.G. Rit. Electroencephalogram and visual evoked potential generation in a mathematical model of coupled cortical columns. *Biological Cybernetics*, 73:357–66, February 1995.

[2] T. Bohlin. *Interactive System Identification: Prospects and Pitfalls.* Springer-Verlag, 1991.

[3] T. Bohlin. Derivation of a 'designer's guide' for interactive 'grey-box' identification of nonlinear stochastic objects. *International Journal of Control*, 59:1505–1524, 1994.

[4] A.H. Jazwinski. *Stochastic Processes and Filtering Theory.* Academic Press, 1970.

[5] A. Isidori. *Nonlinear Control Systems.* Springer, 3rd edition, 1995.

[6] B. Liao. *Convergence and Consistency in Maximum Likelihood Identification of Nonlinear Systems, Report TRITA-REG 8908.* Royal Institute of Technology, Sweden, 1989.

[7] L. Ljung. *System Identification, Theory for the user.* Prentice-Hall, 1987.