CONDITIONAL MORPHOLOGY FOR IMAGE RESTORATION

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ABSTRACT

In this paper some properties of conditional morphology for image restoration are presented.
Conditional morphology is a nonlinear method for coupled edge detection and image smoothing and it is derived from the fusion of two nonlinear Bayesian approaches: statistical morphology and the deterministic annealing solution to Markov Random Fields models.
The proposed method resulting from fusing these approaches, is based on Mean Field Theory, which represents a common theoretical framework able to provide a computational simplification to the problem solution together with maintaining the robustness of Bayesian models.
Experimental results are presented showing good performances obtained by the proposed approach on image affected by impulsive multiplicative noise (e.g. speckle noise).

1. INTRODUCTION

Low level image processing usually involves the problem of coupled edge detection and image restoration. Both edge and restored images are often required by higher level processing modules, for example for pattern recognition tasks.
In literature several methods for coupled edge detection and image smoothing are proposed. Among them, Bayesian methods can be classified in deterministic [1] and stochastic [2] methods, depending on the used approached for estimating the solution.
One of the most known deterministic approach was proposed by Geiger and Girosi and it is based on Mean Field Theory (MFT) [1]. MFT is used for generating an equation which involves the variables of the fields to be estimated by modifying a parameter β in a recursive way, according to a continuation method.
This method is proven to be computationally more efficient than stochastic methods, but it remains quite complex from the computational point of view.
Statistical morphology [3] aims at generalizing mathematical morphology (MM) by means of the introduction of a probabilistic model including MM as a particular case; the output of statistical morphology is obtained by applying MFT, which allows one to define filtering operators presenting robustness and computational simplicity characteristics.
In this paper, the probabilistic model presented in [4] is discussed; on the basis of the common theoretical framework given by the MFT, it allows one to extend statistical morphology for solving smoothing and edge detection problems simultaneously. The proposed method is based on the model proposed in [1]: it represents explicitly both intensity and discontinuity variables and it applies MFT for defining operators more general than statistical morphological ones.
Consequently, conditional morphological operators are defined starting from statistical morphology [3] and the deterministic solution to MRFs model [1]. Further, in this paper an analysis of the behaviour of the conditional morphological operators is performed and experimental results are shown.

2. CONDITIONAL MORPHOLOGY

Conditional morphology [4] allows one to perform edge detection and image smoothing simultaneously by means of the fusion of statistical morphology [3] and the MRF based method presented in [1].

2.1 Statistical morphology

Statistical morphology is a new formulation of classic morphology which makes it more similar to the Bayesian approach. Yuille in [3] redefined basic (dilation and erosion) and complex (opening and closing) morphological operators under a probabilistic point of view.
This methodology involves a temperature parameter T (1/β), whose values change the behaviour of the filter: in fact for $T\to\infty$ $(\beta\to 0)$ morphological classic operators can be obtained, while for $T\to 0$ $(\beta\to\infty)$ linear filtering is performed.
In mathematical morphology the elementary operations are dilation and erosion which consist in maximum and minimum operators respectively.
In statistical morphology, dilation and erosion can be defined respectively *winner-take-all* and *loser-take-all* operations which select value in a particular subset (neighbourhood set) of the image. By considering the input pixel values I_i and the SE B_i (Fig.1) shifted on

each pixel i, for a specific pixel i_0 Yuille *et al* defined binary decision units such that $V_{i_0 j}=1$ and $V_{i_0 k}=0$ for j $\neq$ k; this means that j-th input is selected as the winner at the i_0-th site.

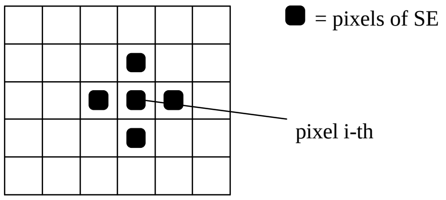

Fig.1: Example of B_i centred on i-th pixel

In this way it is possible to represent each possible solution for the pixel i_0 by means of the set of variables $\{V_{i_0,j}\} = (V_{i_0,1}, V_{i_0,2}, V_{i_0,3}, \ldots\ldots, V_{i_0,n})$ where n is the number of pixels inside SE and with the following constraint $\sum_{j \in SE} V_{i_0,j} = 1$.

By introducing the parameter β and following statistical physics [3], a Gibbs distribution is defined for each configuration:

$$P\left[V_{i_0,k} = 1, V_{i_0,j} = 0, j \neq k\right] = \frac{e^{\beta I_k}}{Z} \qquad (1)$$

where Z represents the partition function $\sum_{j \in SE} e^{\beta I_j}$.

By applying minimum variance estimation to (1), the mean-field equation for statistical dilation is obtained:

$$O_w(I_i) = \sum_{k \in SE} \frac{e^{\beta I_k}}{\sum_{j \in SE} e^{\beta I_j}} I_k \qquad (2)$$

Eq. (2) shows that the solution for statistical dilation at pixel i depends on the parameter β and the grey-level values of pixels that falling inside B_i.

Analogously, Yuille et al. [3] derived the solution for statistical erosion: in this case the solution has the form of eq. (2) in which β assumes negative values.

2.2 Deterministic annealing solution to MRF models

The method proposed in [1] allows one to perform edge detection and image smoothing simultaneously by means of two coupled MRF (F,L): F represents the filtered image intensity field and L represents the presence or the absence of a discontinuity between two neighbouring pixels (line process). More precisely:

- F = $\{F_i$: i=1,.....,MxM$\}$ where F_i represents the image intensity of i-th image pixel (Fig.2); MxM are the number of pixels composing the image *I*;
- h = $\{h_{i,m} \in [0,1]$: i=1,....,MxM$\}$ where $h_{i,m}$ represents the horizontal line process between i-th and m-th neighbour pixels (Fig.2);
- v = $\{v_{i,r} \in [0,1]$: i=1,....,MxM$\}$ where $v_{i,r}$ represents the vertical line process between i-th and r-th neighbour pixels (Fig.2).

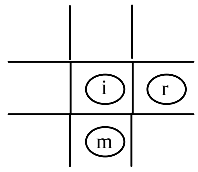

Fig. 2

The input/output relation is a conditional probability density $P_\beta(F, L, / I)$; according to the Bayes theorem, it is possible to write:

$$P_\beta(F, L / I) = \frac{P_\beta(I / F, L) \cdot P(F, L)}{P(I)} \qquad (3)$$

where $P_\beta(I / F, L)$ is a term that represents the noise in the image and depends on the image acquisition.

P(F,L) represents a priori knowledge on the image (e.g. piecewise linear structure) and in this case it moulds the discontinuities in the image by means of the difference between the values (F) of neighbouring pixels. In particular in [1] was used the Weak -Membrane model:

$$P(F, L) = \frac{1}{C} e^{\sum_i \left[\alpha(F_i - F_{i+1})^2 (1 - L_{i,i+1}) + \gamma L_{i,i+1}\right]} \qquad (4)$$

where α and γ are positive parameters that favour the smoothing or preservation of the discontinuities.

Applying minimum variance estimations Geiger & Girosi [1] obtained the following mean-field equation for the line process:

$$\bar{L}_{i,i+1} = \frac{1}{1 + exp\left[\beta\left(\gamma - \alpha\left(F_i - F_{i+1}\right)^2\right)\right]} \qquad (5)$$

The mean field equation for the field F depends on noise parameters and it was obtained by introducing a gradient descent algorithm [1].

2.3 Conditional morphological basic operators

In this paper we modify the statistical morphological model introduced by Yuille *et al.*[3] by taking into account the discontinuities L; it can be performed by introducing a term representing the a priori knowledge about discontinuities in statistical morphology models.

In statistical morphology [3] the value F_i depended on the intensity values of pixels j falling inside B_i, that is the SE centred on pixel *i*, and on parameter β (eq. (2)); in the proposed approach it also takes into account the discontinuities between pixel *i* and neighbour pixels (pixels r and m in Fig.2). Since this dependence is

modelled by means of a set of conditional probabilities, the approach is defined as *conditional morphology*.
The basic conditional morphological operation, erosion and dilation, are obtained by following the same technique presented in [3]: the possible solution for F_i is selected among the pixels of B_i. This concept is formalized by introducing binary decision unit $V_{i,j}$, as in statistical morphology, and a conditional probability $P(F,h,v/I)$. According to Bayes Theorem, we can write:

$$P\left[F_i, h_{i,m}, v_{i,r} / I\right] = \sum_{V_i^J} P\left[F_i, h_{i,m}, v_{i,r} / V_i^J I\right] \cdot P\left[V_i^J / I\right] \qquad (6)$$

where V_i^J represents a possible solution configuration and $P\left[V_i^J / I\right]$ is the same of eq. (*1*).
The dependence of solution from the discontinuities is contained in $P\left[F_i, h_{i,m}, v_{i,r} / V_i^J, I\right]$; in particular for each site j $\in B_i$, we evaluate the interaction among I_j (possible solution) and all possible solutions for F_r and F_m. The F_r and F_m values are chosen in two new neighbourhood sets R_r and M_m, that have the same shape of B_i, but they are centred, respectively, in r and m.
By considering the same approach of [4] it is possible to write:

$$F_i = \sum_{j\in B_i} w_j I_j \qquad (7)$$

with

$$w_j = \frac{\sum_{\{p,k\}} c_{jpk}}{\sum_{j\in B_i} e^{\beta I_j} \sum_{\{p\in B_r, k\in B_m\}} c_{jpk}} \qquad (8)$$

where

$$c_{jpk} = exp\left[\beta\left(I_p - I_k\right)\right] \cdot \left\{ exp\left[\beta\left[2\gamma - \alpha\left(\left(I_j - I_p\right)^2 + \left(I_j - I_k\right)^2\right)\right]\right] + \right.$$

$$\left. + exp\left[\beta\left[\gamma - \alpha\left(I_j - I_p\right)^2\right]\right] + exp\left[\beta\left[\gamma - \alpha\left(I_j - I_k\right)^2\right]\right] + 1 \right\} \qquad (9)$$

with $I_p \in B_r$ and $I_k \in B_m$.
It is possible to notice that weights depend on the values falling inside SEs centred on pixels *i, p* and *m*.
By applying the same method we can derive the solutions for $v_{i,r}$ and $h_{i,m}$:

$$v_{i,r} = \frac{\sum_{j\in B_i} \left(e^{\beta I_j} \sum_{k\in B_m} c_{j,k} \sum_{p\in B_r} e^{\beta I_p} \right)}{\sum_{j\in B_i} e^{\beta I_j} \sum_{\{p\in B_r, k\in B_m\}} c_{jpk}} \qquad (10)$$

$$h_{i,m} = \frac{\sum_{j\in B_i} \left(e^{\beta I_j} \sum_{p\in B_r} c_{j,p} \sum_{k\in B_m} e^{\beta I_k} \right)}{\sum_{j\in B_i} e^{\beta I_j} \sum_{\{p\in B_r, k\in B_m\}} c_{jpk}} \qquad (11)$$

where $c_{j,d} = e^{\beta I_d} \cdot \left\{ e^{\beta\left[\gamma - \alpha(I_j - I_d)^2\right]} + 1 \right\}$ with d={k or p}.
Eqs. (9), (10), (11) represent the outputs of ***conditional dilation*** if β>0. By considering negative β values, they represent the outputs for ***conditional erosion***.

3. COEFFICIENT ANALYSIS

In this section an analysis of the behaviour of the weights in (7) is performed to evaluate the influence of the line process on morphological operators.

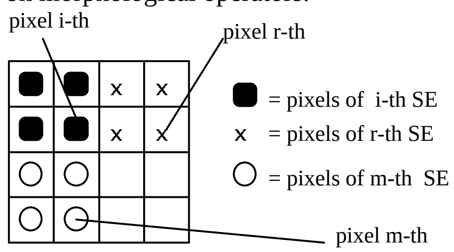

Fig.3

Conditional dilation is here considered by using the model represented in Fig.3.
The behaviour of the conditional morphological operators strongly depends on the parameter β. Two special cases are considered: $\beta \to 0$ and $\beta \to \infty$.

- $\beta \to 0$
Conditional dilation behaves as a linear average, in fact the weights w_j assume the same value $\forall\, j \in B_i$.
- $\beta \to \infty$
From eqs. (8) and (9), it is possible to notice that the output F_i assumes the value of the winner pixel $j \in B_i$ such that w_j is maximized. The winner selection depends on the pixel configuration that is presented in input; more precisely the selection depends on the pixel values in B_i, B_r and B_m and their interactions (i.e. their gradients).
By defining

$$(I_j - I_p)^2 = \Delta I_{jp} \quad \text{and} \quad (I_j - I_k)^2 = \Delta I_{jk}$$

it is possible to individuate the following situations for analyzing the behaviour of conditional morphology.

- *High discontinuity between pixels j and r and between pixels j and m*

$$\begin{cases} \Delta I_{jp} >> \dfrac{\gamma}{\alpha} \\ \Delta I_{jk} >> \dfrac{\gamma}{\alpha} \end{cases} \qquad \forall j \in B_i, \forall p \in B_r, \forall k \in B_m$$

In this case, the behaviour of the weights is the following:

$$w_j \xrightarrow[\beta\to\infty]{} \frac{\sum_{p\in B_r}\sum_{k\in B_m} exp\left[\beta(I_j + I_p + I_k)\right]}{\sum_{j\in B_i}\sum_{p\in B_r}\sum_{k\in B_m} exp\left[\beta(I_j + I_p + I_k)\right]} \qquad (12)$$

As $I_j >> I_p$ and $I_j >> I_k$, it is possible to write:

$$w_j \to \frac{exp(\beta I_j)}{\sum_{j\in B_i} exp(\beta I_j)}$$

This means that if β assumes high values, a statistical morphological dilation is performed and if $\beta \to \infty$ a classical dialtion is performed.

- *Low discontinuity between pixels j and r and between pixels j and m*

$$\begin{cases} \Delta I_{jp} << \dfrac{\gamma}{\alpha} \\ \Delta I_{jk} << \dfrac{\gamma}{\alpha} \end{cases} \qquad \forall j \in B_i, \forall p \in B_r, \forall k \in B_m$$

In this case, the analyzed pixels fall within a uniform region. The behaviour of the weight is described by means of the following expression:

$$w_j \xrightarrow[\beta\to\infty]{} \frac{\sum_{p\in B_r}\sum_{k\in B_m} e^{\left[\beta(I_j+I_p+I_k)\right]} \cdot e^{\left[2\gamma-\alpha(\Delta I_{jp}+\Delta I_{jk})\right]}}{\sum_{j\in B_i}\sum_{p\in B_r}\sum_{k\in B_m} e^{\left[\beta(I_j+I_p+I_k)\right]} \cdot e^{\left[2\gamma-\alpha(\Delta I_{jp}+\Delta I_{jk})\right]}} \qquad (13)$$

From eq. (13), it is possible to observe that the conditional morphological operator behaves as a statistical dilation if: $\Delta I_{jp} \cong \Delta I_{jk} \cong 0$ or $I_j + I_p + I_k + \alpha(\Delta I_{jp} + \Delta I_{jk}) >> 2\gamma$.

- *Presence of an outlier in* B_i

$$I_j^* >> I_j \qquad \forall j \in B_i$$

Within this situation, it is possible to individuate two particular configurations: presence of a uniform region with an outlier in B_i or presence of an outlier in B_i homogeneous with pixels in B_p and B_k.

The first case is described by means of the following conditions:

$$\begin{cases} \Delta I_{jp} << \dfrac{\gamma}{\alpha},\ \Delta I_{jk} << \dfrac{\gamma}{\alpha} & \forall j \neq j^*, j \in B_i, \forall p \in B_r, \forall k \in B_m \\ \Delta I_{jk}^* > \dfrac{\gamma}{\alpha}, \Delta I_{jk}^* > \dfrac{\gamma}{\alpha} & j = j^* \in B_i, \forall p \in B_r, \forall k \in B_m \end{cases}$$

The weights assume the following values:

$$w_j^* \xrightarrow[\beta\to\infty]{} \frac{\sum_{p\in B_r}\sum_{k\in B_m} e^{\beta(I_j+I_p+I_k)}}{\sum_{j\in B_i} e^{\beta(I_j)} \sum_{p\in B_r}\sum_{k\in B_m} c_{jpk}} \qquad (14)$$

$$w_j \xrightarrow[\beta\to\infty]{} \frac{\sum_{p\in B_r}\sum_{k\in B_m} e^{\beta(I_j+I_p+I_k)} e^{\beta(2\gamma)}}{\sum_{j\in B_i} e^{\beta(I_j)} \sum_{p\in B_r}\sum_{k\in B_m} c_{jpk}} \qquad (15)$$

From eqs. (14), (15) it is possible to notice that j^* is solution for pixel *i* if

$$I_j^* > max(I_j) + 2\gamma$$

This expression means that a high γ value is necessary to cancel the outlier j^*.

The second case is described by means of the following condition:

$$\begin{cases} \Delta I_{jp} > \dfrac{\gamma}{\alpha},\ \Delta I_{jk} > \dfrac{\gamma}{\alpha} & \forall j \neq j^*, j \in B_i, \forall p \in B_r, \forall k \in B_m \\ \Delta I_{jk}^* << \dfrac{\gamma}{\alpha}, \Delta I_{jk}^* << \dfrac{\gamma}{\alpha} & j = j^* \in B_i, \forall p \in B_r, \forall k \in B_m \end{cases}$$

There are edges between pixels *i* and *r* and between pixels *i* and *m*, but there is an outlier in $j \in B_i$.

By following the same approximation used before, it is possible to obtain eq. (15) for w_j and eq. (14) for w_j^*; in this case, j^* is solution for pixel *i* if

$$I_j^* + 2\gamma > max(I_j)$$

This means that it is easy that j^* is the winner because it receives a contribution from the parameter γ.

4. EXPERIMENTAL RESULTS

Experimental results aim at showing that conditional morphology provides better results than statistical morphology and itpresents a good adaptability to real images.

To test the proposed operators, a sequence of conditional openings (conditional erosion followed by conditional dilation) is applied to SAR (Synthetic Aperture Radar) images which are corrupted by an impulsive multiplicative noise called "speckle noise”. According to the simulated annealing [1], at each iteration a higher β value is selected and the output obtained at the previous step is processed.

The final structure of the proposed conditional operator, is shown in Fig. 3.

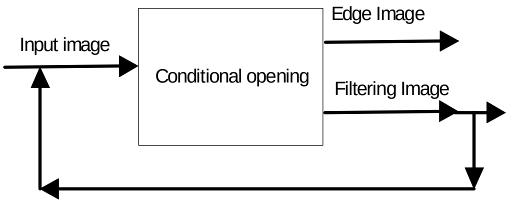

Fig.3: Structure of conditional operator

The following experiment are performed by using a synthetic SAR image (Fig.4) corrupted by a simulated 2-look speckle noise:

- conditional openings (11 iterations, β=0.2, 0.8, 3.2, 12.8, 19.2, 25.8, 51.2, 64.8, 82.4, 104.2, 204.8);
- statistical opening [3], β=104.2; in this case edge were extracted by means of a Sobel operator.

Results are shown in Table 1; it is possible to notice that the better results are obtained both in filtering and edge detection by using the conditional operator.

In Fig. 5 the results obtained applying the conditional opening to real SAR. image are showed.

Analysing Fig. 5 it is possible to notice that the use of coupled probabilistic model in presence of lower noise (4-look noise) improves the result quality. In particular a very good result is obtained in edge detection where the edges are extracted with a particular accuracy (the road in Fig.5c)

5. REFERENCES

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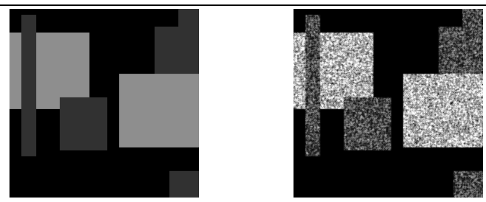
	Lack of edge points	Surplus of edge points	SNR	Noise variance
Corrupted Image			7.82	31.4
Statistical Morphology [3]	0.49	33.87	4.0	48.20
Conditional Morphology	4.58	3.07	11.66	20.44

Table 1: Result of experiments



(a) (c)

Fig.4: a) Synthetic SAR image; b) Synthetic image corrupted by simulated 2-look speckle noise

a) b) c)

Fig. 5: a) SAR image β = 104.2 b) Result from iterative conditional openings with 6 iterations of β (β = 0.8, 3.2, 12.8, 51.2, 104.2, 204.8) c) Edge image obtained using the same steps of case b.