A NONLINEAR CIRCUIT FOR COUPLED IMAGE RESTORATION AND EDGE DETECTION

Marco STORACE, Mauro PARODI and Carlo REGAZZONI

Biophysical and Electronic Engineering Department, University of Genoa, Via Opera Pia 11a, I-16145 Genova, Italy

e-mail: storace@dibe.unige.it parodi@dibe.unige.it carlo@dibe.unige.it

ABSTRACT – A methodological approach to the definition of nonlinear circuits for real-time image processing is presented. The image processing problem is formulated in terms of the minimization of a functional based on the Markov Random Fields (MRFs) theory. The terms of such a functional are related to the co-contents of proper nonlinear multiterminal resistors, thus reporting the minimization process to the achievement of an equilibrium solution in a circuit made up of these multiterminal resistors and of linear capacitors.

Coupled image restoration and edge extraction in the presence of additive gaussian noise are the specific problems addressed in this paper.

1. Introduction

One of the emerging ideas for performing low-level vision tasks in applications with tight real-time requirements is to obtain *analog* image processing by dedicated nonlinear analog circuits [1]. The theoretical efforts regarding these topics are also justified by the advances in VLSI technology, that allow the integration of complex circuit architectures on a single chip.

As shown in [2], many ill-posed early vision problems (e.g., optical flow computation, edge detection) can be solved by minimizing functionals H which are a weighted sum of two terms, i.e., $H = H_1 + \lambda H_2$, according to the regularization theory. The term H_1, called *regularization term*, restricts the class of admissible solutions by introducing suitable a priori knowledge. The term H_2, called *data term*, constraints the solution to remain close to observed data. The weighting coefficient λ controls the compromise between the degree of regularization of the solution and its closeness to the data.

The literature numbers some circuits that minimize, in an analogic and parallel way, functionals structured as H (see, e.g., [3]). The general aspects concerning analogue networks for solving ill-posed variational problems in early vision were extensively discussed in [2,4] starting from Maxwell's minimum heat theorem.

The functionals considered here are built up on the basis of the Markov Random Fields (MRFs) theory. In these functionals, the regularization term takes into account the statistical properties of the solution space and the term λH_2 is formulated starting from the probability distribution of the noise affecting the data [2].

The specific class of MRF image processing models addressed in this work is oriented to coupled image restoration and edge detection in the presence of zero-mean additive white gaussian noise with variance σ^2. The general elements concerning the structure of the functionals are described in [5]. Under the hypothesis that each pixel jk of a given M×N image interacts directly with those contained in its neighbourhood set N_{jk} , the functional H can be written as

$$H^*(X,B) = H_1^*(X,B) + \lambda H_2(X) \qquad (1)$$

with

$$H_1^*(X,B) = \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} \sum_{p,q \in N_{jk}} h(u_{jkpq}, b_{jkpq}) \qquad (2)$$

$$H_2(X) = \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} \left(y_{jk} - x_{jk}\right)^2 \qquad (3)$$

where

$$u_{jkpq} = (x_{jk} - x_{pq})/\Delta \qquad (4)$$

$$h(u,b) = bu^2 + \psi(b) \qquad (5)$$

b_{jkpq} and x_{jk} are elements of the fields B and X, respectively, ψ is a proper function, Δ is a positive constant, $\lambda = 1/2\sigma^2$ is the regularization parameter, and y_{jk} denotes the noisy gray level of the pixel jk before image processing. A method for simplifying the rather complex minimization required by [5] and similar approaches (e.g., [6]) has been recently presented [7]. Such a method allows to reconduce the original functional $H_1^*(X, B)$ to a functional $H_1(X)$ that depends on one set of variables only:

$$H_1(X) = \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} \sum_{p,q \in N_{jk}} \Phi\left(u_{jkpq}\right) \qquad (6)$$

Starting from this basis, we propose a method to synthetize a nonlinear circuit solving the minimization problem. The circuit contains two nonlinear multiterminal resistors $\Re_1$ and $\Re_2$ and linear capacitors, as outlined in Figure 1. Well-defined relationships hold between the MRF functional defining the image processing method and the potential (*co-content*) functions associated to the resistive parts of the nonlinear circuit. These potential functions are homologous to $H_1(X)$ and $\lambda H_2(X)$. This leads to synthetize the multiterminal resistors as corresponding to a specific image processing problem. In the circuit, the stationary values of the capacitors voltages individuate the position of a minimum of the functional H(X).

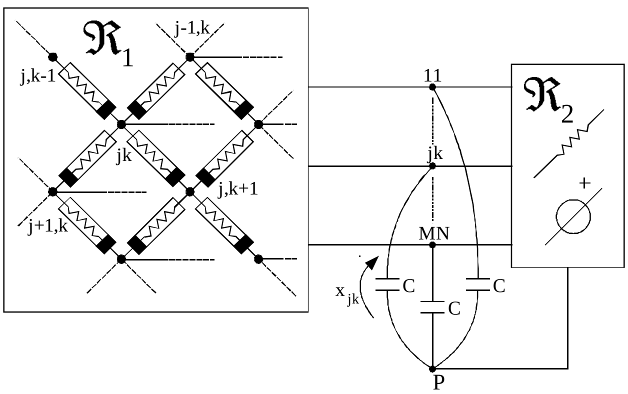


Figure 1

Then, the minimum of the specific MRF functional can be interpreted as an equilibrium state of the circuit.

As sketched out in Figure 1, one of the multiterminal elements is obtained by a grid of identical nonlinear two-terminal resistors. The nonlinear characteristic of these resistors is related to the nonlinear nature of the terms in the functional $H_1(X)$. If, in the considered MRF model, each pixel jk interacts directly only with the four closest ones, then, in the rectangular grid of two-terminal elements, each node is connected directly only to the four closest ones (first-order neighbourhood set).

The analogic image processing performed by the circuit has the following two main advantages:

- the circuit yields a *noise-robust solution* for the considered problem, thanks to the intrinsic properties of the MRF approach;
- the circuit structure implies *parallel processing*.

The computer simulations performed evidence the ability of the circuit to yield real-time solutions for the considered problems. The main dynamic properties of the circuit have been obtained making reference to the properties of the nonlinear resistors characteristic.

2. Definition of the circuit structure

2.1 The resisitive multiterminal $\Re_1$

The M×N nodes of the grid are connected to terminals. The resulting M×N-terminal resistive network is denoted by $\Re_1$. All the resistors are identical and voltage-controlled. The behaviour of $\Re_1$ can be described in terms of the M×N node voltages x_{jk} with respect to a common node P, as shown in Fig. 1. These voltages individuate a *complete set* [8] of variables. For this circuit the global co-content function $\mathcal{G}_1$ is given by the sum of the co-contents of the nonlinear resistors [8]. Denoting by $i = \zeta(v)$ the voltage-controlled characteristic of each resistor, the corresponding co-content term $\hat{\Phi}(v)$ is

$$\hat{\Phi}(v) = \int_0^v \zeta(\rho)\, d\rho \qquad (7)$$

By means of the grid resistors, each inner node jk interacts directly with those individuating the neighbourhood set N_{jk}. In the case of first-order neighbourhood set, for instance, we have $N_{jk} = \{(j,k-1), (j,k+1), (j-1,k), (j+1,k)\}$.

Denoting by jk and pq the terminal nodes of one of these resistors, the pertinent voltage v is given by $x_{jk} - x_{pq}$ and the global co-content $\mathcal{G}_1$ can be thought as a function of the vector $\mathbf{x}$ of the node potentials

$$\mathcal{G}_1(\mathbf{x}) = \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} \sum_{pq \in N_{jk}} \hat{\Phi}(x_{jk} - x_{pq}) \qquad (8)$$

The structure of $\mathcal{G}_1(\mathbf{x})$ is identical to that of the term $H_1(X)$ in expression (6). $\mathcal{G}_1(\mathbf{x})$ corresponds to a given $H_1(X)$ when a proper characteristic $\zeta(v)$ for the nonlinear resistors is individuated.

The relation between the co-content term $\hat{\Phi}$ and the normalized term $\Phi(u)$ (ranging in the interval [-1,0]) in equation (6) can be formulated as follows

$$\hat{\Phi}\left(uV_T\right) = \hat{\Phi}_S\,[\Phi(u) + 1] \qquad (9)$$

The variable $u = v/V_T$ is a dimensionless term defined by taking $\Delta = V_T$ (see expression (4)) and the terms V_T and $\hat{\Phi}_S = \sup_v \hat{\Phi}(v)$ are normalizing constants.

The derivation of the characteristic $i = \zeta(v)$ from a given $H_1^*(X,B)$ can be obtained as follows.

We first observe that the expression (2) for $H_1^*(X,B)$ yields the function $h(u,b)$. The next step consists in obtaining the *even* function $\Phi(u)$ according to the following condition [7]

$$\Phi(u) = \inf_{0 \le b \le M_b} h(u,b) \qquad (10)$$

From a geometrical point of view, $\Phi(u)$ can be interpreted as the infimum of a b-parameterized family of quadratic functions. Then, following [7], the function

$$f(z) = \Phi(\sqrt{u}) = \inf_{0 \le b \le M_b} [bz + \psi(b)] \qquad z, u \in [0,+\infty) \quad (11)$$

is thought as the lower envelope of a one-parameter affine family.

For any given $b \in [0,M_b]$, and for $z \in [0,+\infty)$, the conditions for the function $f(z)$ to be tangent to the b-parameterized family of straight lines $bz + \psi(b)$ are

$$f(z) = bz + \psi(b) \qquad (12)$$

$$\frac{df}{dz} = b \qquad (13)$$

By combining these two conditions, a Clairaut's differential equation [9] for $f(u)$ is obtained:

$$f(z) = z\frac{df}{dz} + \psi\left(\frac{df}{dz}\right) \qquad (14)$$

Instead of considering the general solution (family of straight lines) of equation (14), we refer to the particular solution (envelope of the family of straight lines) that results by eliminating the parameter b from the system of equations

$$\begin{cases} f(z) = zb + \psi(b) \\ z + \dfrac{d\psi(b)}{db} = 0 \end{cases} \qquad (15)$$

Thus, according to definition (11), the even function $\Phi(u)$ is obtained as $f(z^2)$.

As stated in the previous section, the function $\Phi(u)$ can be interpreted as the normalized co-content of each nonlinear resistor inside $\Re_1$. From expressions (7) and (9), then, we find

$$i = \frac{d\hat{\Phi}}{dv} = \frac{d}{dv}\left[\hat{\Phi}_S\ \Phi\left(\frac{v}{V_T}\right) + 1\right] \qquad (16)$$

2.2 The resisitive multiterminal $\Re_2$

The term $\lambda H_2(X)$ in expression (1) can now be considered. As previously stated, this term forces the x_{jk} variables to remain close to the observed data y_{jk} representing the image to be filtered. Being the y_{jk}'s fixed terms, then, also $\lambda H_2(X)$ can be interpreted as the co-content of a proper resistive network $\Re_2$.

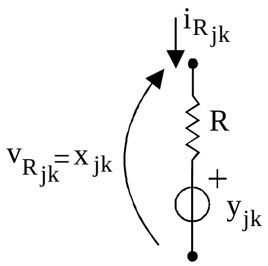


Figure 2

Elementary considerations lead to represent the term $\lambda\ (y_{jk} - x_{jk})^2$ through the two terminal voltage-controlled resistor R_{jk} represented in Fig. 2 (see also [3]). The co-content of R_{jk} can be written as

$$\hat{\Omega}_{jk}(x_{jk}) = \frac{1}{2R}\left[(y_{jk} - x_{jk})^2 - y_{jk}^2\right] \qquad (17)$$

The network $\Re_2$ is made up of M×N resistors R_{jk} , as shown in Fig. 1. Each of them is connected between the reference node P and the pertinent terminal jk of $\Re_1$.

The co-content $\mathcal{G}_2$ (**x**) of $\Re_2$ is given by the sum of the M×N terms $\hat{\Omega}_{jk}$

$$\mathcal{G}_2(\mathbf{x}) = \frac{1}{2R} \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} (y_{jk} - x_{jk})^2 - \frac{1}{2R} \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} y_{jk}^2 \qquad (18)$$

The last term of this expression is constant, so that it does not take part to any minimization process. For this reason, the structure of $\mathcal{G}_2$ (**x**) can be considered equivalent to that of $H_2(X)$.

The term λH_2 in expression (1) can be built up by using the co-content terms $\hat{\Omega}_{jk}$ (see expression (17)).

The comparison between $\lambda H_2(X)$ and the corresponding sum of the dimensionless functions

$$\Omega_{jk}(\mathbf{x}) = \frac{1}{\hat{\Phi}_S} \hat{\Omega}_{jk}(\mathbf{x}) \qquad (19)$$

leads to identify the relative weight λ as

$$\lambda = \frac{1}{2R\hat{\Phi}_S} \qquad (20)$$

In the case of null-mean gaussian white noise with variance σ^2, the maximum a posteriori criterion [7] leads to assign the regularization parameter λ the value $1/2\sigma^2$. Then, the second term in equation (1) can be written as

$$\frac{1}{2\sigma^2} \sum_{\substack{j=1,\dots,N \\ k=1,\dots,M}} (y_{jk} - x_{jk})^2 \qquad (21)$$

Comparing this sum with that originating by the Ω_{jk} and ignoring the constant term (see expressions (17), (18) and (19)), we obtain

$$R = \frac{\sigma^2}{\hat{\Phi}_S} \qquad (22)$$

that directly relates the circuit parameter R to the variance of the noise model through the reference power $\hat{\Phi}_S$.

3. An example

In order to simulate the behaviour of the complete nonlinear circuit, we define the lower and upper limits for the x_{jk}'s voltages as x_{min} and x_{max} , respectively. Then we take the scaling relation

$$x_{jk} = f_{jk} \frac{x_{max} - x_{min}}{255} + x_{min} \qquad (23)$$

between the gray levels $f_{jk} \in [0,255]$ and the corresponding voltages x_{jk}.

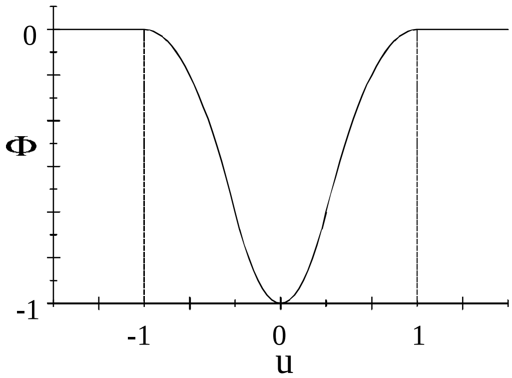


Figure 3

Fig. 3 shows a classical example of Φ(u) corresponding to functions h(u,b) that allow image segmentation and edge detection. As it can be simply verified, the corresponding co-content $\hat{\Phi}$ can be obtained by choosing for the nonlinear resistors of $\Re_1$ the piecewise linear characteristic i = ζ(v) of Fig. 4. In this case, $\hat{\Phi}_S = I_0 V_0 / 2$.

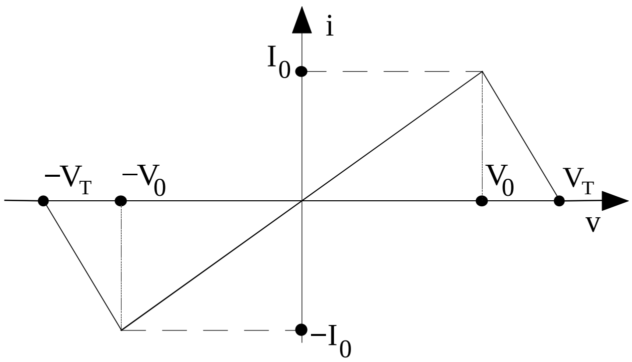
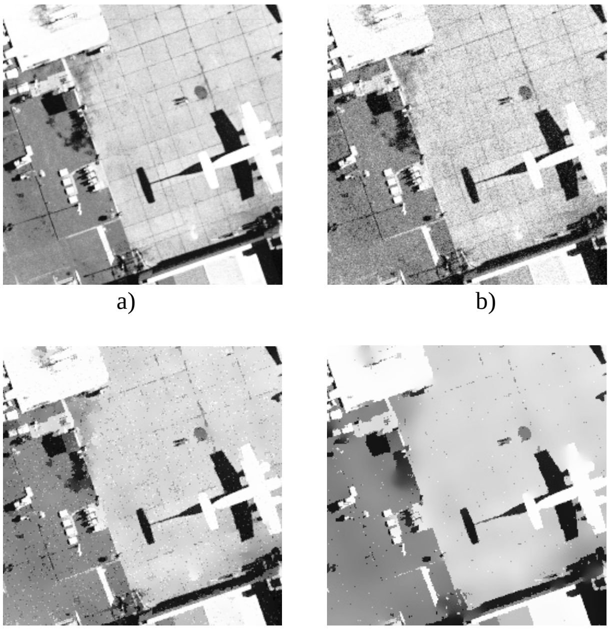


Figure 4

The slope V_0 / I_0 of the central region in Fig. 4 multiplied by C can be arbitrarily taken as a rough reference term τ for the definition of the time scale at which the circuit works.

As an example, a circuit defined by N = M = 256, $V_T = 2V_0 = 0.8V$, $I_0 = 0.8mA$, C = 1nF, $v_{min} = 0$ and $v_{max} = 10V$ is simulated.



a) b) c) d)

Figure 5

Figure 5a shows the original (uncorrupted) image. Fig. 5b shows the image to be processed, obtained by corrupting the image of Fig. 5a by gaussian noise $G(\mu,\sigma)$, with $\mu = 0$ and $\sigma = 15$. Starting from the initial capacitor voltages $v_{jk}(0)$ generated through expression (23), the circuit simulation gives, after few multiples of τ, the steady-state values corresponding to the image in Fig. 5c (first-order model). The image in Fig. 5d is the result obtained by extending the neighbourhood set to eight pixels (particular case of second-order model). In particular, four other nonlinear resistors with a characteristic $i = \zeta(v)/2$ connect each pixel jk to the four pixels {j-2,k ; j+2,k ; j,k-2 ; j,k+2} too. As expected (see, e.g., [7]), the first-order model is very efficient at detecting first-order discontinuities, while the second-order model is more efficient at reconstructing the basic geometric structure of the original intensity surfaces.

References

[1] C. Koch and B. Mathur, "Neuromorphic vision chips," *IEEE Spectrum*, **33**, No. 5, 38-46 (1996).

[2] T. Poggio, V Torre, and C. Koch, "Computational vision and regularization theory," *Nature*, **317**, 314-319 (1985).

[4] T. Poggio and C. Koch, "Ill-posed problems in early-vision: from computational theory to analogue networks", *Proc. R. Soc. Lond. B*, **226**, 303-323 (1985).

[5] D. Geman and S. Geman, “Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images,” *IEEE Transactions on PAMI*, **PAMI-6**, No. 6, 721-741 (1984).

[6] D. Geiger and F. Girosi, “Parallel and deterministic algorithms for MRF's: surface reconstruction,” *IEEE Transactions on PAMI*, **PAMI-13**, No. 5, 401-412 (1991).

[7] D. Geman and G. Reynolds, "Constrained restoration and the recovery of discontinuities," *IEEE Transactions on PAMI*, **PAMI-14**, No. 3, 367-383 (1992).

[3] J.G. Harris, C. Koch, and J. Luo, "A two-dimensional analog VLSI circuit for detecting discontinuities in early vision," *Science*, **248**, 1209-1211 (1990).

[8] L.O. Chua, “Stationary principles and potential functions for non-linear networks,” *J. Franklin Inst.*, **296**, No. 2, 91-114 (1973).

[9] N. Piskunov, *Differential and Integral Calculus*, MIR Publishers, Moskow, 1977.