ON A DIGITAL HEARING AID WITH RECRUITMENT OF LOUDNESS COMPENSATION AND ACOUSTIC ECHO CANCELLATION

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Abstract. **This paper describes a digital hearing aid realized in the frequency domain that compensates for recruitment of loudness and cancels acoustic echos. In contrast to conventional systems which are based on a noise-probe signal, our echo canceller is adapted using only the available (e.g. speech) input signal. The main problems caused by a *nonlinear* feedforward filter, for compensating recruitment of loudness, are discussed using analytical results of the steady state behavior of the closed-loop hearing-aid system. The proposed solutions have been implemented and tested on a dummy behind-the-ear (bte) hearing-aid device.**

Keywords— **Digital Hearing Aid, Adaptive Echo Cancellation, Recruitment of Loudness Compensation, Frequency-Domain Filtering**

1 INTRODUCTION

FEEDBACK is a common problem in hearing aid systems. The maximum stable gain is limited by the feedback. Hearing-impaired persons with loudness recruitment have a reduced range between the sound pressure levels corresponding to threshold and discomfort. As a result, the effective dynamic range of a hearing-impaired person is compressed. It often varies markedly with frequency.

In our paper, we will discuss a hearing-aid device with an adaptive echo canceller, as proposed in [2], together with a nonlinear feedforward filter for compensating recruitment of loudness both working in the frequency-domain as depicted in Fig. 1.

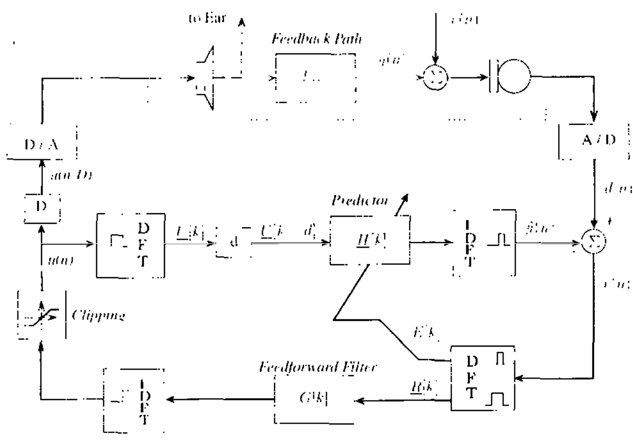

Fig. 1. Hearing aid system with an adaptive echo canceller and a nonlinear feedforward filter.

Note that no probe signal is used in predicting the echo signal. The microphone picks up $d(n) = v(n) + y(n)$, the actual input signal $v(n)$ superimposed with the feedback signal $y(n)$. Including a D/A- and A/D-converter, microphone and loudspeaker, the feedback path is assumed to be modeled by a FIR filter $h(.)$ of length N together with some delay D.

Our realization of predictor and feedforward filter is based on a lapped frequency transform using the so-called *overlap-save* technique [1]. Thereby samples at time index n are collected in blocks of length $2N$ (with an overlap of N samples) yielding for the available input signal $d(n)$ (and equivalently for the loudspeaker signal $u(n)$) $\underline{d}[k] = [d(kN), d(kN+1) \dots d((k+2)N-1)]^T$ with k the block index and T denoting transpose. In estimating $y(n)$, our predictor replaces the linear convolution $u(n-Nd) * \hat{h}(n)$ by the cyclic convolution $\hat{\underline{y}}[k] = \underline{u}[k-d] \circledast \hat{\underline{h}}[k]$, with $\hat{\underline{h}}[k] = [\hat{h}_0[k], \dots, \hat{h}_{N-1}[k], 0, \dots, 0]^T$ a $2N$-point vector obtained by zero padding[1].

By introducing the $2N \times 2N$-point Fourier matrix $\boldsymbol{F}$ with elements $F_{kl} = 1/2N \cdot \exp(-j(2\pi/(2N))kl)$ where $j = \sqrt{-1}$, we get for $\underline{d}[k]$ the appropriate frequency-domain vector $\underline{D}[k] = \boldsymbol{F}\underline{d}[k]$. Capital letters will be used equivalently to describe the frequency transform of the remaining $2N$-point vectors. The cyclic convolution is now written as $\hat{\underline{Y}}[k] = \boldsymbol{U}[k-d]\hat{\underline{H}}[k]$ resulting in a simple multiplication in the frequency domain with the diagonal matrix $\boldsymbol{U}[k]$ having $\underline{U}[k]$ on its diagonal.

The last N samples of $\hat{\underline{y}}[k]$ remain undistorted and hence are used to build up the echo-reduced input signal $r(n) = d(n) - \hat{y}(n)$ (In Fig. 1 this is illustrated with an appropriate window.) Its corresponding block vector is given in the time-domain by

$$\underline{r}[k] = \boldsymbol{p}(\underline{d}[k] - \hat{\underline{y}}[k]) + \boldsymbol{q}(\underline{d}[k-1] - \hat{\underline{y}}[k-1]), \tag{1}$$

with the $2N \times 2N$ projection matrices defined by

$$\boldsymbol{p} := \begin{pmatrix} 0_N & 0_N \\ 0_N & \boldsymbol{I}_N \end{pmatrix}, \quad \boldsymbol{q} := \begin{pmatrix} 0_N & \boldsymbol{I}_N \\ 0_N & 0_N \end{pmatrix}. \tag{2}$$

In principle, the predictor $\hat{\underline{H}}[k]$ is adapted to the feedback path $\underline{H}[k]$ in the frequency domain using a power-normalized least mean square (LMS) algorithm. It is given by

$$\hat{\underline{H}}[k+1] = \hat{\underline{H}}[k] + \boldsymbol{\mu}[k] \cdot \boldsymbol{U}^H[k-d]\underline{E}[k], \tag{3}$$

with the diagonal step-size matrix $\boldsymbol{\mu}[k]$ normalized by $\mu_i[k] = \mu_0/\mathcal{E}[|U_i[k]|^2]$ [2]. As proposed in [5], the error vector is given in the time domain by $\underline{e}[k] := \boldsymbol{p}\underline{r}[k]$ and in

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[1] For simplicity we will not use the partitioning technique as proposed e.g. in [2].

the frequency domain by

$$\underline{E}[k] = \boldsymbol{P}\underline{R}[k]. \tag{4}$$

With the help of (1) the error signal can be related to the echo-reduced signal by $\underline{r}[k] = \underline{e}[k] + \boldsymbol{q}\underline{e}[k-1]$ and equivalently in the frequency domain by

$$\underline{R}[k] = \underline{E}[k] + \boldsymbol{Q}\underline{E}[k-1]. \tag{5}$$

The matrices $\boldsymbol{Q} = \boldsymbol{F}\boldsymbol{q}\boldsymbol{F}^{-1}$ and $\boldsymbol{P} = \boldsymbol{F}\boldsymbol{p}\boldsymbol{F}^{-1}$ denote the appropriate projection matrices (2) in the frequency domain. We note that, based on relation (5), $\underline{R}[k]$ can be computed from $\underline{E}[k]$ with N additions only [2].

The feedforward filter $\underline{G}[k]$ compensates for recruitment of loudness and is proposed to be built up by additive weighting of a set of arbitrarily defined bandpass filters $\underline{G}_l$ in the frequency domain, i.e.

$$\underline{G}[k] := \sum_l W_l[k] \cdot \underline{G}_l\,. \tag{6}$$

If the impulse response of each bandpass filter $\underline{G}_l$ is at most of length N, any superposition has the same length. Hence, $\underline{G}[k]$ is guaranteed to produce no aliasing if the filter is time-independent or varies slowly with time.

In a real hearing-aid device, the loudspeaker has a saturation level. To prevent our linear predictor from having to estimate a nonlinear feedback path, we introduce a clipping device as depicted in Fig. 1 with a clipping level below the saturation level of the loudspeaker.

With no clipping, the predictor input vector can finally shown to be

$$\underline{U}[k] = \boldsymbol{P}\boldsymbol{G}[k]\underline{R}[k] + \boldsymbol{Q}\boldsymbol{G}[k-1]\underline{R}[k-1]. \tag{7}$$

We will continue in Section 2 with an analysis of the steady-state behavior in the mean-square sense of our hearing aid system. Based on the obtained results, we will then discuss in Section 3 the problems due to the nonlinear feedforward filter. Section 4 will finally present our solutions which have been verified successfully with the help of a dummy bte hearing-aid device.

2 ANALYSIS OF THE STEADY-STATE BEHAVIOR

The following analysis assumes a stable system in its steady state. Feedback path and feedforward filter are assumed to be time-independent, i.e., $\underline{G}[k] = \underline{G}$, $\underline{H}[k] = \underline{H}$. Furthermore, to be mathematically tractable, the analysis is based on the following assumptions:

(a1) The input vectors $\underline{v}[k]$, $\underline{v}[k+1], \cdots$ are mutually uncorrelated, i.e. $\mathcal{E}\left[\underline{v}[k]\underline{v}^H[k+i]\right] = 0$, for $i \neq 0$.
(a2) The vectors $\underline{u}[k]$, $\underline{u}[k+1], \cdots$ remain mutually uncorrelated. In addition, its frequency transform $\underline{U}[k]$ constitutes with its components an *uncorrelated complex Gaussian process*, i.e. $\mathcal{E}[U_i^*[k]U_j[k]] = 0$, for $i \neq j$.

The following properties follow from (a1) and (a2):

(p1) The error vectors $\underline{E}[k]$, $\underline{E}[k+1], \cdots$ are mutually uncorrelated.
(p2) $\underline{\tilde{H}}[k]$ has uncorrelated components for $k \to \infty$.

With $\underline{\tilde{H}}[k] := \underline{H} - \underline{\hat{H}}[k]$ and assuming predictor and feedback path to have the same delay, i.e. $Nd = D$, we start by rewriting the error vector (4) as

$$\underline{E}[k] = \boldsymbol{P}\underline{V}[k] + \boldsymbol{P}\tilde{\boldsymbol{H}}[k]\underline{U}[k-d] \tag{8}$$

with the matrix $\tilde{\boldsymbol{H}}[k]$ representing $\underline{\tilde{H}}[k]$, analog to $\boldsymbol{U}[k]$. With (a1) and $\underline{\tilde{H}}[k]$ being uncorrelated to $\underline{U}[k-d]$ (follows from (3) with (a2)), we find for the error correlation matrix

$$\begin{aligned}\mathcal{E}\left[\underline{E}[k]\underline{E}^H[k]\right] &= \boldsymbol{P}\mathcal{E}\left[\underline{V}[k]\underline{V}^H[k]\right]\boldsymbol{P}\\ &+ \boldsymbol{P}\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\mathcal{E}\left[\underline{U}[k-d]\underline{U}^H[k-d]\right]\tilde{\boldsymbol{H}}^H[k]\right]\boldsymbol{P}.\end{aligned} \tag{9}$$

In proceeding, we replace (5) in (7). Using the identity

$$\boldsymbol{P}\boldsymbol{G}\boldsymbol{Q}\underline{E}[k] + \boldsymbol{Q}\boldsymbol{G}\underline{E}[k] = \boldsymbol{G}\boldsymbol{Q}\underline{E}[k] \tag{10}$$

the predictor input vector becomes

$$\underline{U}[k] = \boldsymbol{P}\boldsymbol{G}\underline{E}[k] + \boldsymbol{G}\boldsymbol{Q}\underline{E}[k-1] + \boldsymbol{Q}\boldsymbol{G}\boldsymbol{Q}\underline{E}[k-2] \tag{11}$$

and, with the help of (p1), its correlation matrix

$$\begin{aligned}\mathcal{E}\left[\underline{U}[k-d]\underline{U}^H[k-d]\right] &= \boldsymbol{P}\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\boldsymbol{P}\\ &+ \boldsymbol{G}\boldsymbol{Q}\mathcal{E}\left[\underline{E}[k-d-1]\underline{E}^H[k-d-1]\right]\boldsymbol{Q}^H\boldsymbol{G}^H\\ &+ \boldsymbol{Q}\boldsymbol{G}\boldsymbol{Q}\mathcal{E}\left[\underline{E}[k-d-2]\underline{E}^H[k-d-2]\right]\boldsymbol{Q}^H\boldsymbol{G}^H\boldsymbol{Q}^H\,.\end{aligned} \tag{12}$$

According to assumption (a2) off-diagonal elements can be neclected. For the diagonal elements, Eq. (12) can be simplified with the help of $\mathcal{E}\left[\underline{E}[k]\underline{E}^H[k]\right] = \mathcal{E}\left[\underline{E}[k+i]\underline{E}^H[k+i]\right]$ for all i (steady-state analysis). It results in

$$\mathcal{E}\left[\underline{U}[k-d]\underline{U}^H[k-d]\right] = 2\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\,. \tag{13}$$

yielding for Eq. (9)

$$\begin{aligned}\mathcal{E}\left[\underline{E}[k]\underline{E}^H[k]\right] &= \boldsymbol{P}\mathcal{E}\left[\underline{V}[k]\underline{V}^H[k]\right]\boldsymbol{P}\\ &+2\boldsymbol{P}\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\tilde{\boldsymbol{H}}^H[k]\right]\boldsymbol{P}\,.\end{aligned} \tag{14}$$

With $\mathrm{tr}\{.\}$ being the trace operator, we now write for the so-called *mean squared error* (MSE)

$$J[k] := \frac{1}{N}\sum_i |E_i[k]|^2 = \frac{1}{2N^2}\mathrm{tr}\left\{\mathcal{E}\left[\underline{E}[k]\underline{E}^H[k]\right]\right\}\,. \tag{15}$$

The MSE can be written as $J[k] = J_{\text{ex}}[k] + J_{\text{min}}$, consisting of a predictor-dependent so-called *excess* MSE $J_{\text{ex}}[k]$ and a term J_{min} dependent only on $v(n)$. With $J_{\text{min}} = 1/2N^2 \times \mathrm{tr}\{\boldsymbol{P}\mathcal{E}[\underline{V}[k]\underline{V}^H[k]]\,\boldsymbol{P}\}$, we obtain for the excess MSE

$$J_{\text{ex}}[k] = \frac{1}{N^2}\cdot\mathrm{tr}\left\{\boldsymbol{P}\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\tilde{\boldsymbol{H}}^H[k]\right]\boldsymbol{P}\right\} \tag{16}$$

and finally with $\mathrm{tr}\{A \cdot B\} = \mathrm{tr}\{B \cdot A\}$ and (p2)

$$J_{\mathrm{ex}}[k] = \frac{1}{N^2}\,\mathrm{tr}\left\{\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\tilde{\boldsymbol{H}}^H[k]\right]\right\}$$
$$= \frac{1}{2N^2}\sum_i \mathcal{E}\left[|E_i[k-d]|^2\right]|G_i|^2\,\mathcal{E}\left[|\tilde{H}_i[k]|^2\right]. \quad (17)$$

With the misadjustment of the predictor defined by $\mathcal{M} = J_{\mathrm{ex}}[k]/J_{\min}$ $(k \to \infty)$, we obtain the following result describing the mean-square steady-state behavior of our system:

Proposition 1: For a stable system according to Fig. 1, the misadjustment $\mathcal{M}$ fulfills

$$\frac{\mathcal{M}}{\mathcal{M}+1} = \frac{J_{\mathrm{ex}}[k]}{J[k-d]} = \frac{\sum_i \mathcal{E}\left[|E_i[k-d]|^2\right]|G_i|^2\,\mathcal{E}\left[\left|\tilde{H}_i[k]\right|^2\right]}{\sum_i \mathcal{E}\left[|E_i[k-d]|^2\right]} \quad (18)$$

for the steady state $(k \to \infty)$.

The quality of the echo-reduced signal $r(n)$ is measured with the misadjustment $\mathcal{M}$. Keeping it constant (by means of an appropriate step-size matrix $\boldsymbol{\mu}[k]$, see [2]) means that an increase in the gain of the feedforward filter $\boldsymbol{G}$ can only be compensated for by reducing the coefficient mismatches $\mathcal{E}\left[|\tilde{H}_i[k]|^2\right]$, if possible. Note that a stable system has a steady state with $J_{\mathrm{ex}}[k] < J[k-d]$, or $\mathcal{M} < \infty$.

3 PROBLEMS OCCURING DUE TO A NONLINEAR TIME-VARIANT FEEDFORWARD FILTER

In order to compensate for recruitment of loudness, the frequency-dependent dynamic range of the signal has to be adapted to the requirements of the impaired person. A typical compression function for a given frequency band is depicted in Fig. 2. Such a compression requires high gain for small input levels and low gain for high input levels resulting in a nonlinear weighting functions $W_l[k]$ for the corresponding bandpass filters $\underline{G}_l$ in Eq. (6). The use of such compression functions leads to three main problems:

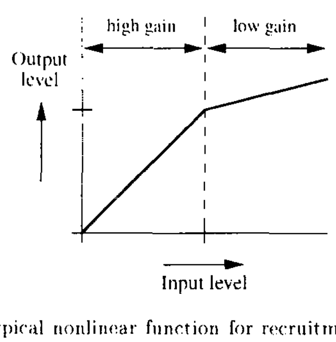

Fig. 2. Typical nonlinear function for recruitment of loudness compensation.

The first problem arises due to the non stationarity of the input signal (e.g. speech). The input signal might switch between low and high level causing successive high and low gains in weighting the bandpass filters. If the gain difference between successive blocks is too large the fast gain change in the feedforward filter will cause *audible artefacts* in the loudspeaker signal due to aliasing effects.

The next problem concerns the echo canceller. Assuming the error vector $\underline{E}[k]$ to be white, according to Eq. (18), a predictor aiming at a fixed misadjustment essentially requires the sum $\sum_i |G_i|^2\mathcal{E}\left[|\tilde{H}_i[k]|^2\right]$ to be constant. Hence, a low-gain feedforward filter will allow for a larger predictor mismatch $\underline{\tilde{H}}[k]$ if compared to a high-gain filter. Hence, a sudden change from a low-gain to a high-gain filter will cause severe artefacts, since the predictor needs some time to reduce its mismatch $\underline{\tilde{H}}[k]$. The result might even be an instable system.

The third problem concerns instabilities of the hearing-aid system caused from outside: A sudden change of the feedback path might result in an open-loop gain "$G\tilde{H}$" that produces an unstable hearing-aid system. By minimizing $\underline{E}[k]$, the predictor tries to stabilize the system. However, if the open-loop gain of the system is too large, the predictor might be to slow to cope with the increase of $\underline{E}[k]$. Such a situation increases the input level of the feedforward filter until the nonlinear region of the compression function is reached. The gain is automatically reduced until an open-loop gain of exactly 1 is obtained resulting in a sustained oszillation. If the hearing-aid system is in this state, we have to react in some way, e.g. by reducing the overall gain.

4 SOLUTIONS TO THE DISCUSSED PROBLEMS

4.1 Aliasing due to large gain changes

From experience we observed that a sudden increase of a low-level input signal to a high-level signal combined with a corresponding gain reduction does not result in any audible artefacts. If in contrast a high-level signal is suddenly decreased to some low-level, a corresponding increase of the gain results in very disturbing artefacts. From this we propose to limit the relative increase of the gain. Due to the temporal post-masking effect [4] such a limitation does not produce any audible artefacts. With $W_l^{(d)}[k]$ the desired weight of the l'th bandpass, the actually used weight $W_l[k]$ is obtained as

$$W_l[k+1] = \begin{cases} \alpha\, W_l[k] & (W_l^{(d)}[k+1] > \alpha\, W_l[k]) \\ \beta\, W_l[k] & (W_l^{(d)}[k+1] < \beta\, W_l[k]) \\ W_l^{(d)}[k+1] & \text{else} \end{cases} \quad (19)$$

With the appropriate choice of α $(\alpha > 1)$ and β $(0 < \beta \ll 1)$ one does not hear any artefacts and the feedforward filter still compresses and compensates for recruitment of loudness. β can be about 5% and α should not be much larger than 1.1 [6].

4.2 Large low-gain mismatch of the predictor

If a predictor aiming at a constant misadjustment is used, we observe the above-mentioned problem: Switching from a high-gain to a low-gain feedforward filter will cause the predictor to increase its mismatch $\underline{\tilde{H}}[k]$. A forthcoming increase to a high-gain filter will then end up in a mismatch $\underline{\tilde{H}}[k]$ which causes annoying artefacts.

To overcome this problem, we propose to reduce – or even stop – the adaptation for low-gain feedforward filters. Even if stopped, the predictor has still sufficiently long adaptation time intervals if the input signal is properly scaled for the dynamic range of the hearing aid device.

4.3 System instabilities – An open-loop-gain measure

A potential unstable system, e.g caused by a sudden change of the feedback path, reacts as discussed above: it uses the gain-reduction mechanism of the feedforward filter in order to reduce the open-loop gain to 1 resulting in a sustained oscillation. To detect this undesired system condition, we propose to use $K_0[k] := \sqrt{J_{\text{ex}}[k]/J[k-d]}$, a measure of the open-loop gain. We use an available estimate that is a close lower bound of $K_0[k]$. The obtained analytical result is given by the following proposition:

Proposition 2: For the system according to Fig. 1, $K_0[k]$ and the estimate defined by

$$\hat{K}_0[k] := \frac{\sum_i \mathcal{E}[|E_i[k]E_i^*[k-d]|]}{\sum_i \mathcal{E}\left[|E_i[k-d]|^2\right]} \tag{20}$$

fulfill for the steady state ($k \to \infty$) $\hat{K}_0[k] \leq K_0[k]$.

To prove it, we first substitute (7) in (8) which yields with (5)

$$\begin{aligned} \underline{E}[k] = \boldsymbol{P}\underline{V}[k] &+ \boldsymbol{P}\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\boldsymbol{G}\left(\underline{E}[k-d] + \boldsymbol{Q}\underline{E}[k-d-1]\right) \\ &+ \boldsymbol{P}\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\boldsymbol{G}\left(\underline{E}[k-d-1] + \boldsymbol{Q}\underline{E}[k-d-2]\right) . \end{aligned} \tag{21}$$

With (p1), (21) results in

$$\mathcal{E}\left[\underline{E}[k]\underline{E}^H[k-d]\right] = \mathcal{E}\left[\boldsymbol{P}\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\boldsymbol{G}\underline{E}[k-d]\underline{E}^H[k-d]\right] . \tag{22}$$

To proceed, we consider the *Cauchy-Schwarz* inequality:

$$\left(\mathcal{E}\left[\underline{y}^H\underline{x}\right]\right)^2 \leq \mathcal{E}\left[\underline{y}^H\underline{y}\right] \cdot \mathcal{E}\left[\underline{x}^H\underline{x}\right] . \tag{23}$$

We use a much tighter bound by using a vector $\underline{y}$ that depends with its component signs on $\underline{x}$. It is given by

$$\left(\sum_i \mathcal{E}[|y_i x_i|]\right)^2 \leq \sum_i \mathcal{E}\left[|y_i|^2\right] \cdot \sum_i \mathcal{E}\left[|x_i|^2\right] \tag{24}$$

With $\underline{x} := \underline{E}[k-d]$ and $\underline{y} := \boldsymbol{P}\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\boldsymbol{G}\underline{E}[k-d]$, (24) allows to write with (22)

$$\begin{aligned} &\left(\sum_i \mathcal{E}[|E_i[k]E_i^*[k-d]|]\right)^2 \leq \\ &\text{tr}\left\{\boldsymbol{P}\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\boldsymbol{P}\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\right]\right\} \\ &\qquad \times \sum_i \mathcal{E}\left[|E_i[k-d]|^2\right] . \end{aligned} \tag{25}$$

It leads with

$$\begin{aligned} &\text{tr}\left\{\boldsymbol{P}\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\boldsymbol{P}\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\right]\right\} < \\ &\qquad \text{tr}\left\{\boldsymbol{P}\mathcal{E}\left[\tilde{\boldsymbol{H}}[k]\boldsymbol{G}\mathcal{E}\left[\underline{E}[k-d]\underline{E}^H[k-d]\right]\boldsymbol{G}^H\tilde{\boldsymbol{H}}[k]\boldsymbol{P}\right]\right\} \end{aligned} \tag{26}$$

and Eq. (17) for $k \to \infty$ to the claimed proposition $\hat{K}_0[k] \leq K_0[k]$. □

Simulations have shown $\hat{K}_0[k]$ to be very accurate in estimating $K_0[k]$, especially in the vicinity of one. Hence, we propose to use it for controlling the maximum gain of the feedforward filter in order to prevent an unstable hearing aid device.

In our DSP implementation – driving microphone and loudspeaker of a behind the ear (bte) hearing aid device – the gain is reduced by 50% if $\hat{K}_0[k]$ surpasses 0.9. The gain is linearly recovered within 2s in order to allow sufficient time for the predictor to find an accurate estimate of the actual feedback path. In doing so, we obtained an implementation which has no audible artefacts and which runs gains more than 20 dB above the critical gain. The used predictor length was $N = 64$ for a sampling rate of 16 kHz and a delay $d = 3N$ of 12ms.

5 CONCLUSIONS

We have presented a steady state analysis of a closed-loop hearing-aid system incorporating an adaptive echo canceller and a nonlinear feedforward filter for compensating recruitment of loudness both realized in the frequency domain. Based on this analysis, three main problems have been discussed and solutions proposed and verified by means of a real-time DSP implementation. An estimation of the open-loop gain has been proposed to be used in controlling the stability of the overall system. Using only the available speech input signal (no injected white-noise probe signal) for the echo canceller, the device runs more than 20 dB above the critical gain without any audible artefacts.

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