ADAPTIVE SYSTEM IDENTIFICATION WITH TAP-ASSIGNMENT IN OVERSAMPLED AND CRITICALLY SAMPLED SUBBANDS

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Abstract. Adaptation of the tap profile in subband adaptive system identification problems can further enhance the efficient use of computational resources if implemented on a DSP with an otherwise too tight benchmark performance. Here, we derive a generalization of previous work to extend tap-assignment algorithms to a new class of oversampled filter banks with non-uniform bandwidths and different subsampling ratios. We compare efficiency and adaptation results for this approach to the critically sampled case and a fullband identification with same complexity.

1. INTRODUCTION

For the identification of long impulse responses, as eg. found in acoustic echo control problems, often the achievable model may be limited to a truncated solution due to the computational benchmark of a DSP. While IIR filters offer lower complexity but cannot adequately match the nature of the problem [1], FIR subband adaptive filters (SAF) appeal through the advantages of reduced complexity, increased time representation of the filter taps, and its parallel structure. The subband approach further allows to exploit the spectral characteristics of the system to be identified by appropriately distributing the computations over different subbands, eg. dedicating longer filters and more computations to the low frequency range when modelling acoustic systems [2, 5].

At a given complexity, we can perform the assignment of taps adaptively. This idea was first introduced by [6], with refinements in [12, 14]. However, their use of undecimated subband signals counteracts the task of efficiency. In [15] we have demonstrated the improved system representation over fullband adaptive filtering when employing critical decimation, and presented a simplified adaptive tap profile algorithm.

Critically sampled filter banks (FB) are free of redundancy and can be designed to have perfect reconstruction (PR) property, but suffer from aliasing between bands and require the use of cross terms to compensate for the leakage of information at the band edges when processing of the subband signals is intended [7]. In contrast, a new breed of near PR,

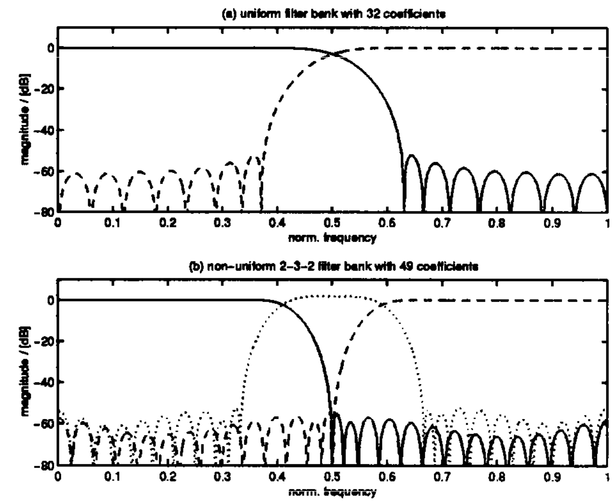

Fig. 1: Filter magnitude responses of (a) a uniform FB (QMF filter 32C from [3]) and (b) a non-uniform OSFB with unequal bandwidth filters [8]; decimation ratios of the three resulting subbands are $\mathbf{r} = [2,3,2]^T$. Stronger branched filter banks can be achieved by iteration of these filters [15].

oversampled filter banks (OSFBs) described in [9] avoids aliasing between bands – thus making cross terms obsolete – and in-band aliasing by the use of non-uniform bandwidth filters with different decimation factors, as shown in Fig. 1(b).

In the following, we apply tap-assignment to normalized least-mean-square (NLMS) adaptive filtering in subbands produced by this new class of OSFBs by generalizing our approach in [15] to account for unequal subsampling ratios, that result in different complexities and time representations for taps in different subbands.

2. SUBBAND ADAPTIVE FILTERING

We want to apply adaptive filters to the subband signals that we yield through decomposition of input and desired signal by an analysis filter bank. With critically sampled subbands, cross terms with a fixed and an adaptive part are required [7], which compensate for leaked information and allow to adapt with otherwise alias-distorted parts of the signal. Using the OSFB approach described in [8, 9], aliasing is avoided and thus cross terms can be neglected. Fig. 1 shows the magnitude plots for both types of filter banks. The overall error can be calculated via a synthesis bank, which for our OSFBs is symmetrical to the analysis bank by design [9], thus fulfilling conditions for a tight frame decomposition in [4].

A reconstruction of the equivalent fullband response of the subband adaptive system may be accomplished off-line by sending an impulse through analysis bank, adapted subband filters (in case of critical sampling also the cross terms), and the synthesis bank. It can be shown that the reconstruction error is bounded by the distortion function of the filter bank [13, 15].

3. MEASURES FOR COMPLEXITY AND TIME-REPRESENTATION

A digital filter running in a subband at a lower rate compared to the fullband system is characterized by two facts: (i) one filter tap now refers to a longer sampling period, thus resulting in an increased *time representation* compared to a fullband filter tap, and (ii) the filter is operated at a lower rate, thus reducing *complexity*. To handle these quantities for an SAF with different subsampling ratios, we need to introduce common measures.

Let $\mathbf{r} \in \mathbb{N}^M$ with entries r_m holding the subsampling ratios in M subbands, which can be factored as $\mathbf{r} = a\tilde{\mathbf{r}}$, $a \in \mathbb{N}$, such that $\tilde{\mathbf{r}}$ is free of any common integer divisor. We then define:

- *Equal Complexity Unit (ECU).* An SAF in the mth band of length N_m has the same complexity as a fullband filter of length $\tilde{N}_m = N_m/\tilde{r}_m$[1]. We define 1 ECU as $\frac{1}{\tilde{r}_m}$, ie. the smallest integer common to all subsampling ratios. This allows to compare complexity and exchange taps in ECU between different subbands.
- *Equivalent Fullband Time Representation (EFTR).* The EFTR of a filter of length N_m in the mth band is approximately given by $\hat{N}_m = r_m N_m$ fullband taps, as the sampling period in the mth band is increased by a factor of r_m compared to the fullband[2].

In the following, ECU and EFTR quantities are characterized by $\tilde{(\cdot)}$ and $\hat{(\cdot)}$, respectively.

For judgement of complexity reduction, let us assume a constant DSP system benchmark of B multiply-accumulate (MAC) operations per fullband sampling period. The complexity C_b for a filter bank calculation is given through inspection by

$$C_b = L \cdot \sum_{d=1}^{D} \left(\sum_{m=0}^{M-1} \frac{1}{r_m} \right)^d , \tag{1}$$

where it is assumed that the filter bank is organized in a tree structure by D times iterating the filters in Fig. 1 with M channels, filter length L, and subsampling ratios $\mathbf{r}$.

The total complexity of adaptive filtering using the NLMS algorithm (eg. [10]) referring to one fullband sampling period is given by

$$C_a = \sum_{m=0}^{M-1} \frac{2N_m + 2}{r_m} = \frac{2}{a} \sum_{m=0}^{M-1} \left(\tilde{N}_m + \frac{1}{\tilde{r}_m} \right). \tag{2}$$

Therefore, the overall number of available ECUs is

$$\tilde{N} = \sum_{m=0}^{M-1} \tilde{N}_m = \frac{a}{2}(B - 3C_b) - \sum_{m=0}^{M-1} \frac{1}{\tilde{r}_m}. \tag{3}$$

The $\tilde{N}$ ECUs can be exchanged freely between bands to alter the tap profile. We next look at two possible extrema.

Uniform EFTR. If all SAFs have same fullband time representation $\hat{N}_{\min} = \hat{N}_i = \hat{N}_j, \forall i, j \in \{0, M-1\}$, each filter requires $\tilde{N}_m = \hat{N}_{\min}/(r_m \tilde{r}_m)$ ECUs. Insertion in (2) yields

$$\hat{N}_{\min} = \tilde{N} \cdot \left(\sum_{m=0}^{M-1} \frac{1}{r_m \tilde{r}_m} \right)^{-1} . \tag{4}$$

Concentrated EFTR. Here, either the plant or the input signal's energy is confined to the frequency range covered by a single subband, to which all taps can be dedicated. We use an average decimation ratio $\bar{r}$ to compensate for possible variations in $\mathbf{r}$:

$$\hat{N}_{\max} = \tilde{N} \cdot \frac{\bar{r}^2}{a}, \text{ with } \bar{r} = \sum_{m=0}^{M-1} r_m . \tag{5}$$

Examples for the extrema in time representation are shown in Fig. 2 for different benchmarks, with results stated relative to the length $N = \frac{1}{2}B - 1$ of a fullband NLMS adaptive filter. The assumed filter banks are iterations of the filters characterized in Fig. 1. The range of EFTRs spanned between $\hat{N}_{\min}$ and $\hat{N}_{\max}$ provides the motivation for an adjustable distribution of the adaptive filter weights.

4. TAP-PROFILE ADAPTATION

Global error minimization. Let input signal and observation noise in each subband, x_m and n_m, be

[1] Note that the critically sampled case is included with $\tilde{r}_m = 1 \forall m \in \{0, M-1\}$.

[2] We neglect the delay that has to be placed in the desired path of the SAF, growing longer with increased bank depth D [11].

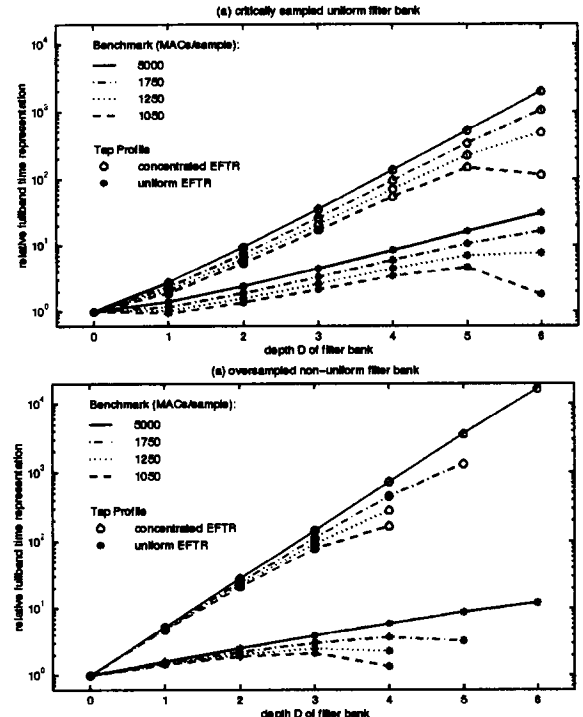

Fig. 2: Relative time representation extrema $\hat{N}_{\min}/N$ and $\hat{N}_{\max}/N$ for different benchmarks in dependency on the filter bank depth D for (a) critically sampled uniform subbands and (b) oversampling non-uniform subbands relative to a fullband adaptive filter.

uncorrelated and wide sense stationary. With the expectation operator $\mathcal{E}\{\cdot\}$, the minimum of the mean-squared error (MSE) in the mth band can be expressed as [15]

$$\mathcal{E}\{e_m^2\}\big|_{\min} = \sum_{k=N_m}^{\infty} w_{\mathrm{opt},m}^2(k)\cdot\sigma_{x_m}^2 + \sigma_{n_m}^2, \qquad (6)$$

where $\mathbf{w}_{\mathrm{opt},m}$ are the optimum weights of the response to be identified, projected into the mth subband. Thus, the residual error in (6) depends on the coefficients that cannot be identified due to truncation by a too short adaptive filter, weighted by the variance of the input signal $\sigma_{x_m}^2$ and biased by the noise variance $\sigma_{n_m}^2$.

The relation between the energy in the subband signals and the synthesized signal is governed by a fixed gain, as the critically sampled filter banks allow application of Parseval's theorem, while the non-uniform OSFBs in [8] implement a tight frame decomposition. Assuming steady state performance of the adaptive filters, global and subband MSE are related by

$$\mathcal{E}\{e^2\} = \sum_{m=0}^{M-1} \alpha_m \mathcal{E}\{e_m^2\}, \qquad (7)$$

where $\sum_m \alpha_m = A^{-1}$, with A being the frame bound and oversampling ratio, holds if the l_2-norm of the analysis filters is unity. Minimizing the global MSE thus requires

$$\alpha_i\mathcal{E}\{e_i^2\} \stackrel{!}{=} \alpha_j\mathcal{E}\{e_j^2\} \stackrel{!}{=} \min \forall i,j\in\{0,M{-}1\}. \qquad (8)$$

The adjustment of the filter lengths N_m can be interpreted as a constraint problem, where the profile should be optimized to yield a minimum global error, while the overall computational complexity of the system must remain constant.

Algorithm. Our proposed profile adaption is based on a constant pool of ECUs, which are suitably distributed over the subbands. Every P fullband samples (with P being a common multiple of $\tilde{r}$) a profile adaptation step [12, 15]

$$\tilde{N}_m(k{+}1) = \tilde{N}_m(k) - \Delta\left(1 - \frac{M c_m(k)}{\sum_{m=0}^{M-1} c_m(k)}\right),$$

is performed, where Δ ECUs are withdrawn from each SAF and re-distributed according to an appropriate criterion $\mathbf{c}\in\mathbb{R}^M$ with entries c_m. Different from [15], here we employ a fractional book keeping rather than an integer record, as for widely deferring decimation rates large ECU blocks had to be exchanged, which would result in coarse assignment and required large P causing slower convergence. A post-processing stage has to ensure that subband filters have a minimum number of taps remaining which at least equal the delay in the desired path [11, 15].

The tap-assignment error criterion $\mathbf{c}$ can take on different forms. Firstly, it may be set equal to estimates of the subband MSEs [6] following (6). Secondly, a more robust but bias criterion [12], with simplifications in [15] and a direct equivalence to the first summand in (6), may be employed, which estimates the truncation error from the power of the last coefficients in each filter, multiplied by the power of the input signal. Both criteria now need to be weighted by α_m in the mth band to account for the implemented frame decomposition.

Finally, the length of the SAFs is calculated from the adapted ECU quantities and rounded according to $N_m = \lfloor \tilde{N}_m \tilde{r}_m \rfloor$.

Example. For a comparison of adaptive filtering in fullband, 2 band uniform, and a non-uniform 2-3-2 subband system, we discuss a system identification example with a system benchmark $B = 1250$ MACs/sample of an IIR system with dominant poles at $\Omega = 0.5$ and $\Omega = 0.9$. Fig. 3 shows the reconstructed equivalent fullband models, with truncation and truncation errors listed in Tab. 1, while the adaptation of the tap profile is depicted in Fig. 4 for the two subband cases. Clearly, the oversampled system yields a better exploitation of computational resources through the omission of cross-terms — at

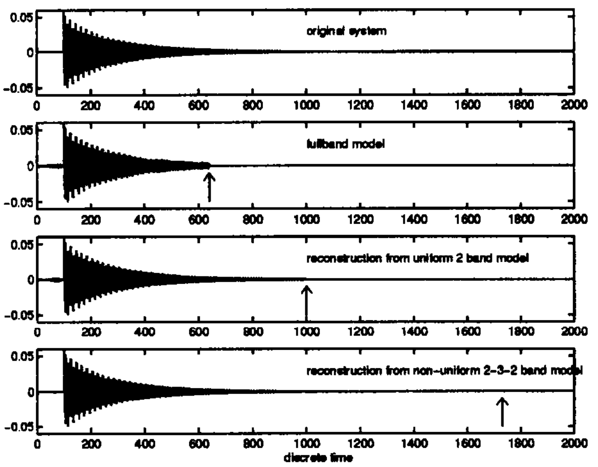

Fig. 3: Comparison between a fullband model, and reconstructed models from adaptive filters 2 uniform and 2-3-2 non-uniform subbands for a system benchmark $B = 1250$ MACs/sample. Arrows indicate truncation.

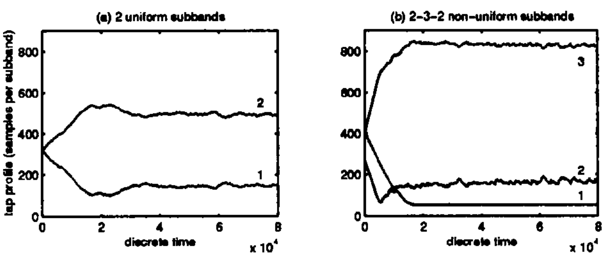

Fig. 4: Example for profile records for (a) uniform and (b) oversampled non-uniform subband schemes.

least for low bank depth D. Note that the error of the reconstructed equivalent fullband model for the non-uniform bank case closely approaches the bank distortion of -53.4dB.

5. CONCLUSIONS

We have derived a generalization for tap profile adaptation to accommodate for a new kind of filter banks [8, 9] with non-uniform bandwidths and different subsampling ratios. Using these non-uniform OSFB for adaptive filtering in combination with tap-assignment has the potential of substantially improving the accuracy of system identification of very long impulse responses over critically sampled systems. Besides convergence issues discussed in [9], it also has advantages over critically sampled subbands through the omission of cross-terms, which can save computations and makes implementations less complex, although this is bought at the expense of different, potentially prime, decimation rates in a system.

Although formulae have been specifically derived for the use with NLMS subband adaptive filters, the results can easily be transfered to other $\mathcal{O}(N)$ algorithms.

method	EFTR	error
fullband	624	-21.6 dB
2 band uniform	$\approx$ 1000	-29.0 dB
non-uniform 2-3-2 band	$\approx$ 1720	-49.4 dB

Tab. 1: Reconstruction error $\|w_{\rm opt} - w_{\rm recon}\|_2^2$ and model length for given example.

6. REFERENCES

[1] Gudvangen S and Flockton SJ: "Modeling of Acoustic Transfer-Functions for Echo-Cancelers". *IEE-Proc Vision Image Sig Proc*, 142(1):47–51, 1995.

[2] Abbott P: "Acoustic Echo Cancelling in Video Telephony and Conferencing on a Single DSP". in *Proc DSP UK II*, London, Vol. 2, 1995.

[3] Crochiere RE and Rabiner LR: *Multirate Digital Signal Processing*. Prentice Hall, 1983.

[4] Cvetković Z and Vetterli M: ""Oversampled Filter Banks"". To appear in *IEEE Trans SP*.

[5] Diethorn E: "Perceptually Optimum Adaptive Filter Tap Profiles for Subband Acoustic Echo Cancellers". in *Proc ICSPAT*, Boston, pp. 290–293, 1995.

[6] Ma Z, Nakayama K, and Sugiyama A: "Automatic Tap Assignment in Sub-Band Adaptive Filter". *IEICE Trans Comms*, E76-B(7):751–754, 1993.

[7] Gilloire A and Vetterli M: "Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experiments and Applications to Acoustic Echo Cancelation". *IEEE Trans SP*, SP-40(8):1862–1875, 1992.

[8] Harteneck M, Paez-Borallo JM, and Stewart RW: "A Filterbank Design for Oversampled Filter Banks without Aliasing in the Subbands". Proc. TFTS'97, Warwick, UK, 1997.

[9] Harteneck M, Paez-Borallo JM, and Stewart RW: "An Oversampled Subband Adaptive Filter Without Cross Adaptive Filters". Proc. IWAENC'97, London, 1997.

[10] Haykin S: *Adaptive Filter Theory*, Prentice Hall, 2nd edition, 1991.

[11] Kellermann W: "Analysis and Design of Multirate Systems for Cancellation of Acoustical Echoes", in *Proc. IEEE ICASSP*, New York, 1988, vol. 5, pp.2570–2573.

[12] Sugiyama A and Hirano A: "A Subband Adaptive Filtering Algorithm with Adaptive Intersubband Tap-Assignment". *IEICE Trans Fund*, E77-A(9):1432–1438, 1994.

[13] Vaidyanathan PP: *Multirate Systems and Filter Banks*. Prentice Hall, 1993.

[14] Vukadinovic M and Aboulnasr T: "A Study of Adaptive Intersubband Tap Assignment Algorithms from a Psychoacoustic Point of View". in *Proc ISCAS*, Atlanta, vol. 2:65–68, 1996.

[15] Weiß S, Sörgel U, and Stewart RW: "Computationally Efficient Adaptive System Identification in Subbands with Intersubband Tap Assignment for Undermodelled Problems". in *Proc 30th Asilomar Conf Sign Sys Comp*, Vol. 2:818-822, Monterey, 1996.