Feedforward Algorithms in Active Noise Control

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ABSTRACT

In the paper a new approach to stability problem of feedforward control with LMS identification is presented. A modification improving speed of LMS identification is introduced. Then new narrowband noise cancellation algorithms are described and their results are compared to results obtained by FIR and IIR filters. The idea of the algorithms proposed is extended to broadband noise cancellation. In the last part of the paper sampling with various frequencies is considered and concept of multirate signal processing is proposed as a solution for extending of the attenuation band.

1. INTRODUCTION

In the literature there are a lot of approaches to noise cancellation based on feedforward, feedback, or hybrid (combined both of these techniques) control. It is natural that for narrowband sounds, such as simple tones, feedforward control is sufficient to achieve satisfactory cancellation results [1]. Moreover feedforward control, in contrary to feedback control, works out control value prior to appearing the error value. This implies considerably better attenuation. But astonishingly, feedforward is almost only performed via finite impulse response filters.

This paper mainly aims at presenting new feedforward concepts for narrowband and broadband noise cancellation on the background of FIR and IIR algorithms and a modification of LMS algorithm.

Experimental results were carried out on a real-time plant - personal active hearing protector (see Fig.1) with f_s=2 [kHz]. Linearity of the plant is assumed for further considerations. Attenuation factor (*AF*) is calculated in the band of <100,5000> [Hz] with f_s=40 [kHz] by Solartron Schlumberger Analyser [3].

2. FEEDFORWARD ALGORITHMS FOR ACTIVE NOISE CONTROL

Block diagram of feedforward control is depicted on Fig.2. LMS is used as the identification algorithm. For FIR filters ($F(z^{-1},i) = S(z^{-1},i)$, where $S(z^{-1},i)$ is a z^{-1} polynomial of order dS) control value is calculated as a weighted sum of only reference signal x.

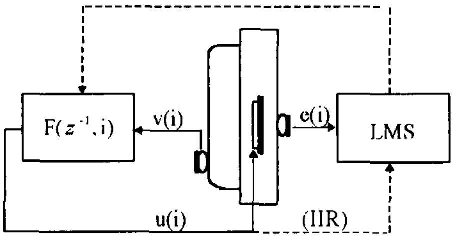

Fig. 1. Personal active hearing protector with feedforward control.

It is specific that FIR filters are perfectly adjusted to the frequency of the signal (see Fig. 2) not matter if the signal is cancelled (400 [Hz]) or not (800 [Hz]).

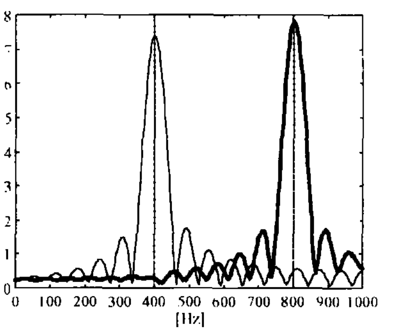

Fig. 2. Moduli of the frequency response of FIR filters adjusted to 400 (thin) and 800 [Hz] (thick).

But for frequencies beyond the attenuation band (*AB*) the filter parameters increase linearly in time. After examining their behaviour, the following relation was noticed:

$$|w|_{Max} > 0.5 \quad \Rightarrow \quad |w| \sim t . \tag{1}$$

As the result control values are only constrained by the hardware. This implies that rectangular-shaped signal is send to the secondary source. Cancellation is then impossible. Thus, a solution is to constrain the parameters. Commonly known from the literature *Leaky LMS* failed to cope with the problem. The parameters became bounded but unfortunately the *AB* was not extended and even the attenuation factor was worse.

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A new modification of the LMS algorithm was proposed. Similarly to normalisation of reference signal, filter parameters are proposed to be normalised. This modification was named *Normalised-W LMS (NWLMS)* and the parameters update equation takes form (2):

$$w(n+1) = w(n) + \frac{\mu x(n)e(n)}{b\,\mathrm{Max}(|w(n)|) + a}, \qquad (2)$$

where a and b denote constant coefficients, adjusted experimentally (e.g. $a=0.05$ and $b=2$). The band was not extended as well, but the speed of convergence was increased about ten times, and the steady state error was diminished, what is extremely important in such application like personal hearing protector. Assuming nullified starting parameters, in the first stage of identification in the denominator only a exists what reveals as increasing of μ 20 times. This reflects in speeding up the algorithm but also increasing the steady state error. During the adaptation process, the norm becomes larger and it reveals as decreasing the step size several times. Finally the steady state error decreases and attenuation improves.

Looking for the reason why the filters diverge for frequencies beyond the attenuation band, an analysis of roots of the filters was carried out. It turned out that they are nonminimumphase outside *AB* (see: Fig. 3) and minimumphase - inside.

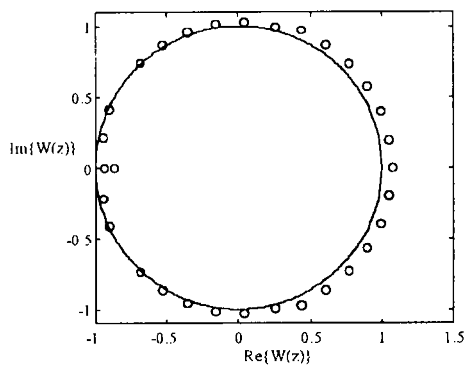

Fig. 3. Zeros (o) of FIR filter adjusted to 800 [Hz].

Feedforward control with FIR filter should ensure stability of the system unconditionally. But if the filter is adaptive any adaptation algorithm uses error signal to update filter parameters. This introduces "artificial" feedback path to the system and leads to instability if the system is nonminimumphase. Taking into account results of spectral analysis (moduli of frequency responses of the filters and power spectral densities of control signals that confirm proper frequency adjustment) the idea of employing spectral factorisation was put forward. This stabilises the system but still requires a phase matching algorithm.

In conclusion it can be summarised that FIR filters reveal the following features:

- in case of identification by LMS, the adaptation converges to the global minimum because the performance surface $E\{e^2(i)\}$ of adaptive FIR filters is always quadratic [4];
- involve high order to achieve required *AF*;
- have very big selectivity for narrowband signals;
- the speed of convergence depends on exciting signal and it increases with the number of parameters;
- do not accumulate quantization errors;
- being fixed, they guarantee stability, but in their adaptive version, an instability can be introduced due to the feedback generated by an adaptation algorithm;
- are able to cancel noise in $AB \in$ <300; 450> [Hz] with $AF = 40$ [dB];
- they perfectly cope with real nonstationarity of amplitude of the noise to be cancelled and with nonstationarity of its frequency up to 50 [Hz].

For IIR filters ($F(z^{-1}) = \frac{S(z^{-1},i)}{R(z^{-1},i)}$, where $R(z^{-1},i)$ is a z^{-1} polynomial of order dR) control value is calculated as a weighted sum of both reference and control signals. IIR filters adjust quite well to frequencies being cancelled but only at the lower limit of their attenuation band (see: Fig. 4).

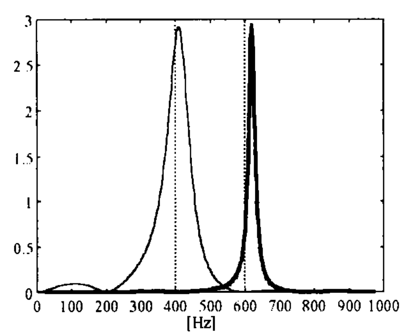

Fig. 4. Moduli of the frequency response of IIR filters adjusted to 400 (thin) and 800 [Hz] (thick).

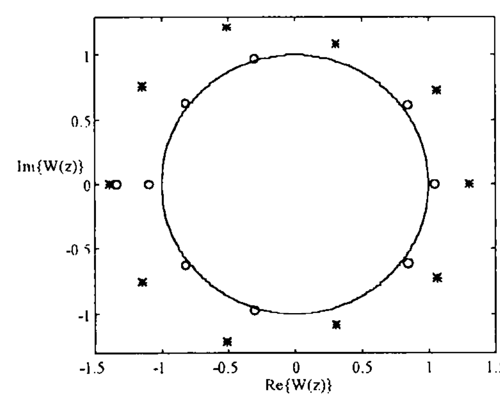

Fig. 5. Zeros (o) and poles (*) of IIR filter adjusted to 400 [Hz].

In conclusion it can be summarised that IIR filters reveal the following features:

- in case of identification by LMS, the adaptation may converge to a local minimum because the performance surface $E\{e^2(i)\}$ of adaptive IIR filters is generally nonquadratic and may be multimodal [4];
- involve smaller order then FIR filters to achieve the same performance;
- have lower selectivity for narrowband signals;
- have very big speed of convergence;
- accumulate quantization errors;
- compensate influence of acoustic feedback from control signal to measured reference signal;
- can introduce instability to the system being both fixed (some poles of the filter may lay outside the unit circle) and adaptive (feedback generated by an adaptation algorithm);
- are able to cancel noise in $\Delta B \in$ <300; 450> [Hz] with ΔF = 40 [dB];
- they perfectly cope with real nonstationarity of amplitude of the noise to be cancelled and they are very poor in case of nonstationarity of its frequency (they can cope with deviations only up to 5 [Hz]).

2.1 PHS

For pure tones better results were achieved when Phase Shifter (PHS) was employed [3]. This adaptive algorithm is based on physical / heuristic approach stating that a sinusoid passing through any linear path is changed only in magnitude and phase. In the system under consideration it refers to reference signal x and control signal u. To achieve noise cancellation in the real plant at observation point e, it is not necessary to perform complicated processing over signal x but only scale it in magnitude and delay in time. Due to continuous character of the real plant, time delays introduced by all its parts are not integer multiples of the sampling period. An algorithm able to model any required phase shift was designed and can be described in the following form:

$$F(z^{-1},i) = z^{-q}\frac{s_i}{1-r_i z^{-1}} = \frac{S(z^{-1},i)}{R(z^{-1},i)}. \tag{3}$$

The operator z^{-q} allows to roughly delay the signal (noise) with accuracy to half the sampling period. Parameter q is evaluated on the basis of minimisation of the sum of squared errors and it is not unique [3]. The filter $\frac{1}{1-r_i z^{-1}}$ ensures correction of the remaining part of the delay. Scale factor s_i added to such a filter ensures amplitude matching of the two signals to be interfered. Both r_i and s_i are identified by LMS algorithm.

PHS reveals the following features [3]:

- its concept is based on physical - not automatic - approach to simple sound cancellation problems;
- is suitable only for narrowband sounds with spectrum concentrated around one frequency;
- ensures global minimum of the performance surface when identified by LMS method;
- having minimum number of parameters - at least an order less then for the other solutions - guarantee comparable attenuation effects;
- extends attenuation band: 250 - 500 [Hz] with sampling rate of 2 [kHz];
- constitutes a wideband filter;
- convergence speed is almost independent of the exciting signal;
- does not accumulate quantization errors;
- ensures attenuation of pure tones up to 40 [dB] (60 [dB] measured in peaks).

2.2 PHS2

This algorithm has had the same origin as PHS and is based on similar concept. The number *2* in its name comes from *two* parameters to be identified (4).

$$F(z^{-1}) = z^{-1}\frac{g_i(b_i + z^{-1})}{1+b_i z^{-1}} = \frac{S(z^{-1})}{R(z^{-1})} \tag{4}$$

The modulus of the frequency response is uniform and the phase can be changed in the range <$-\pi$;0>. For any phase changes only parameter b_i is responsible and for amplitude matching - parameter g_i. It is specific that only two parameters are to be identified and such a filter meets all the requirements to actively cancel any pure tone. Its features are very similar to features of PHS presented above. Attenuation factor reaches values like those obtained by PHS but the band is wider <180;680> [Hz]. Besides, it does not need discrete time delay identification what takes majority of the time. Concluding the results presented above and the analysis of computational burden, PHS2 algorithm seems to be better then the others and even PHS. But on the other hand, in PHS2 phase adjustment is performed only via one parameter and the parameter is responsible for correction of π while in PHS algorithm parameter r_i adjusts the phase only of $\frac{f}{f_s}\pi$ (e.g. for f = 250 [Hz] and f_s = 2000 [Hz], the adjustment is of $\frac{1}{8}\pi$). So the sensitivity of PHS2 is very high (at least four times higher then of PHS) and finally its robustness to nonstationarities is poorer.

3. BROADBAND NOISE CANCELLATION

3.1. Complex tones cancellation

The idea of PHS (as well as PHS2) was extended to broadband noise and was named PHS Banks. Each bank consists of a band-pass filter and a PHS (see: Fig. 6).

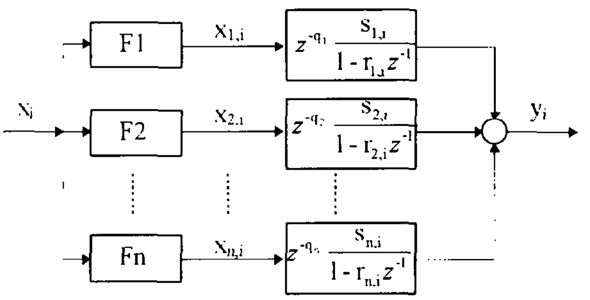

Fig. 6. The block diagram of PHS-Banks algorithm.

A PHS can cope with signal having spectrum not wider then about 40 [Hz], so the filters should be properly designed. They have to have very high selectivity and moduli of the frequency response of neighbouring filters do not have to cross each other in resonance peaks (see: Fig.7). They are suggested to be designed using a least-squares method.

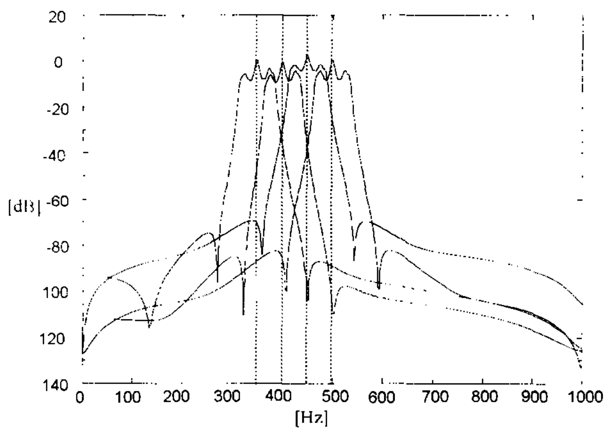

Fig. 7. Moduli of the frequency response of four filters designed for PHS-Banks.

Assuring such constraints, described algorithm is able to attenuate any sound in the whole band up to 40 [dB]. The frequency limit is imposed only by the speed of the signal processor employed. It is very important that all the PHS filters are destined for bands known beforehand. Thus, the discrete time delays can be fixed in advance and do not have to be identified. Finally, for n banks only $2n$ parameters: $s_1, \dots, s_n$, and $r_1, \dots, r_n$ have to be identified (e.g. by LMS) what constitutes the same number of parameters as for PHS2.

3.2. Extension of the attenuation band

Experiments performed with various sampling frequencies (2, 1, 0.5 [kHz]) and feedforward control with PHS or FIR adaptive filters show that for each sampling frequency attenuation bands obtained are adjacent or slightly overlapped, and usually octave (e.g. for PHS: f_s=2[kHz]$\Rightarrow AB \in$ <250;500> [Hz]; f_s=1[kHz]$\Rightarrow AB \in$<150;300>[Hz];f_s=0.5[kHz]$\Rightarrow AB \in$<100; 125>[Hz]). On the basis of these results it was found that varying sampling rate, it is possible to move noise cancellation range along frequency axis. An algorithm converting signal sampled with an arbitrary chosen frequency to signals as if they were sampled with other frequencies is termed *Multirate Signal Processing (MSP)* [2]. So the idea is to sample signals with one frequency and process them in different channels with different rates covering very wide band. For the problem under consideration the MSP system consists of band-pass anti-aliasing filters, down-samplers, adaptive FIR or PHS filters, up-samplers, and low-pass anti-imaging filters. It is noteworthy that adaptive filters implemented as FIR filters are identical with exactly the same parameters at each channel [2] what makes the band-pass anti-aliasing filters very efficient if they are properly designed (e.g. for 17 parameters only 4 multiplications are required) [2]. It was experimentally proved that employing the idea of MSP combined with FIR [2] or PHS, it is possible to cancel any noise in any band. The limits are imposed only by the hardware equipment used (the lower limit is constrained by the pass-band of loudspeakers and the upper limit - by the speed of the signal processor used).

4. CONCLUSIONS

In the paper feedforward control was thoroughly explored. Both narrowband and broadband noise cancellation were considered. New algorithms were presented and stability problems were discussed.

On the basis of the results obtained from the real-world experiment and presented in the paper, and from others mentioned in the literature (e.g. [1]) one can state that feedforward adaptive control is a powerful approach to noise control problems.

5. REFERENCES

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