Complementary N-Band IIR Filterbank Based on 2-Band Complementary Filters

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***Abstract*—A structure and design procedure for complementary N-band IIR filterbanks is proposed. The N-band filterbank has a tree-like structure with cascaded 2-band complementary filters and related all-pass filters. No inverse filterbank is needed and the subband signals are simply added to re-construct the time-domain signal. While no perfect reconstruction is achieved, the filterbank impulse response is all-pass with a well-behaved group delay. The proposed filterbank is suitable for delay-critical audio processing applications. The computational complexity of the proposed IIR filterbanks is significantly lower than that of corresponding FIR filterbanks.**

I. Introduction

When an audio processing or enhancement application is not delay-critical, it is often advantageous to use frame-based processing. The use of a short-time Fourier transform [1], [2] or a downsampled filterbank [3], [4] implies frame-based processing, whereas the frame-size corresponds to the downsampling factor of the filterbank. Compared to sample-wise processing, frame-based processing usually can be implemented with lower computational complexity due to efficient filterbank design and operation of an algorithm in downsampled signal domains. The disadvantage of frame-based processing is the delay which occurs because one has to wait until frame-size input samples arrived before processing the samples.

A filterbank suitable for delay-critical audio processing applications processes the input signal sample-wise. The use of IIR filterbanks as opposed to FIR filterbanks for this application has the advantage of lower computational complexity. We propose IIR filterbanks suitable for audio processing applications, not with a perfect reconstruction property, but with perfect frequency magnitude reconstruction and relatively small and frequency-smooth group delay. Both of these properties are necessary to achieve high audio quality.

The proposed N-band filterbank has the structure of a tree, based on a cascade of 2-band complementary (low- and high-pass) IIR filters. It will be shown that in order to be N-band complementary, all-pass filters, related to the various used 2-band complementary filters, need to be inserted into the tree. No synthesis filterbank is needed and the signal is reconstructed by adding all band signals.

Up to 8-band complementary IIR filterbanks with uniform bands have been proposed in [5]. Additionally, [5] describes the use of synthesis filterbanks for implementation of multi-rate systems. The procedure described here is more flexible in terms of supporting arbitrary frequency decompositions with any number of bands, e.g. for approximating auditory filterbanks. Synthesis filterbanks and multi-rate systems are not treated in this paper.

The paper is organized as follows. Section II reviews 2-band complementary IIR filters suitable for use with the proposed filterbank design. The proposed design of an N-band complementary filterbank is described in Section III. An example of a 12-band complementary filterbank is given in Section IV, including its properties such as magnitude frequency response and group delay. Discussion and conclusions are in Sections V and VI, respectively.

II. Two-Band Complementary IIR Filters

The proposed method considers, as a starting point, a pair of filters which are complementary. The filters F and $\tilde{F}$ are complementary if they satisfy

$$|F(e^{\jmath\omega}) + \tilde{F}(e^{\jmath\omega})| = 1\,, \tag{1}$$

i.e. their sum is an all-pass filter.

Example magnitude responses of two complementary filters are shown in Fig. 1, where the solid line shows the response of a third order Butterworth low-pass filter with a cut-off frequency of 2 kHz and the dashed line shows the corresponding complementary filter.

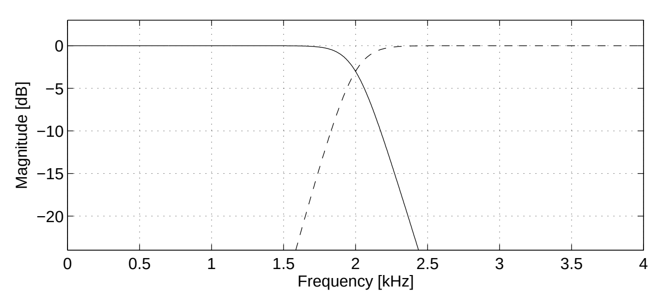

Fig. 1. The magnitude response of a low-pass filter (solid) and its complementary filter (dashed).

A practical way of implementing a pair of complementary filters is to write the filter F as a sum of two all-pass filters [6]. The complementary filter $\tilde{F}$ can then be expressed as the difference between the same all-pass filters. This result is valid for a wide range of digital filters [7] and can be applied to well-known low-pass Butterworth, Chebyshev, and elliptic digital filters of odd orders [8].

III. N-Band Complementary IIR Filters

The proposed method extends the 2-band complementary IIR filters, discussed in Section II, to N-band complementary IIR filterbanks. This is achieved by cascading any number of 2-band complementary IIR filters with arbitrary transition frequencies.

As a first example, a simple case with $N = 3$ bands is considered, illustrated in Fig. 2.

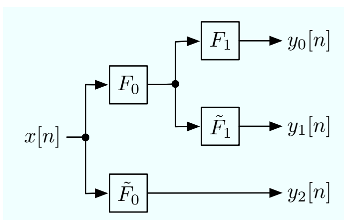

Fig. 2. Example of a 3-band complementary IIR filterbank implemented using a cascade of two 2-band complementary IIR filters.

The transfer function for each band n ($0 \leq n < N$) is denoted as H_n. For the 3-band example, using the filters F_k and $\tilde{F}_k$ with $k = 0, 1$, the transfer functions are

$$\begin{cases} H_0(z) = F_0(z) \cdot F_1(z) \\ H_1(z) = F_0(z) \cdot \tilde{F}_1(z) \\ H_2(z) = \tilde{F}_0(z) \,. \end{cases} \tag{2}$$

The corresponding transfer function of the sum of the sub-bands is

$$\begin{aligned} H(z) &= H_0(z) + H_1(z) + H_2(z) \\ &= F_0(z) \cdot (F_1(z) + \tilde{F}_1(z)) + \tilde{F}_0(z) \,. \end{aligned} \tag{3}$$

This transfer function is not all-pass and thus, as is, the 3-band filterbank is not complementary.

To obtain a 3-band complementary filterbank, the all-pass filter

$$G_2(z) = F_1(z) + \tilde{F}_1(z) \,, \tag{4}$$

is applied to the third band signal, resulting in a filterbank transfer function

$$H(z) = (F_0(z) + \tilde{F}_0(z)) \cdot (F_1(z) + \tilde{F}_1(z)) \,. \tag{5}$$

This transfer function is a product of the sum of two complementary filters, i.e. a product of two all-pass filters, which is all-pass (1). Thus, the 3-band filterbank with the additional all-pass filter G_2 is complementary.

In the following, this result is extended to a filterbank with any number of bands N using any form of cascade of 2-band complementary filters. The cascaded 2-band complementary filters are implemented using filters F_k and $\tilde{F}_k$, with $0 \leq k < K$. The N transfer functions H_n, including additional all-pass filters G_n, should verify the complementary property:

$$\left| \sum_{n=0}^{N-1} H_n(e^{\jmath\omega}) \cdot G_n(e^{\jmath\omega}) \right| = 1 \,. \tag{6}$$

Let $\mathcal{K} = \{0, 1, \ldots, K-1\}$ be the set of indexes used to enumerate the filters F_k and $\tilde{F}_k$. Since the filterbank is a cascade of 2-band complementary filters, the band n ($0 \leq n < N$) transfer function H_n can be written as a product,

$$H_n(z) = \prod_{i \in \mathcal{I}_n} F_i(z) \cdot \prod_{j \in \mathcal{J}_n} \tilde{F}_j(z) \,, \tag{7}$$

where $\mathcal{I}_n$ and $\mathcal{J}_n$ are the two subsets of $\mathcal{K}$ containing the indexes of the filters used for band n. Because of the cascade structure of the proposed filterbank, both subsets $\mathcal{I}_n$ and $\mathcal{J}_n$ cannot be simultaneously empty and their intersection is the empty set, i.e. $\mathcal{I}_n \cap \mathcal{J}_n = \emptyset$. The corresponding all-pass filter G_n that is applied to band n is

$$G_n(z) = \prod_{l \in \mathcal{L}_n} \left(F_l(z) + \tilde{F}_l(z) \right) , \tag{8}$$

where $\mathcal{L}_n$ is $\mathcal{K}$ excluding the union of $\mathcal{I}_n$ and $\mathcal{J}_n$,

$$\mathcal{L}_n = \mathcal{K} \setminus (\mathcal{I}_n \cup \mathcal{J}_n) \,. \tag{9}$$

Thus, the resulting filterbank transfer function, corresponding to the sum of the all-pass (8) filtered subband signals (7), can be written as

$$H(z) = \sum_{n=0}^{N-1} H_n(z) \cdot G_n(z) = \prod_{k=0}^{K-1} \left(F_k(z) + \tilde{F}_k(z) \right) . \tag{10}$$

The right side of this equation implies that H is all-pass, and therefore the described filter cascade with the all-pass filters corresponds to an N-band complementary filterbank.

The proposed tree structure can be further simplified. If all all-pass filters G_n, related to bands involving the same 2-band complementary filters in the tree, include the same all-pass filter term $F_l + \tilde{F}_l$ (8), then this term can be removed from the all-pass filters G_n and moved to the input of these 2-band complementary filters. This retains the same filterbank transfer function, while less all-pass filter terms are used. Based on this insight, the goal is to move all all-pass terms into the filterbank tree such that the minimum number of terms are used. The filterbank band transfer functions can then be written as (7),

$$H_n(z) G_n(z) = \prod_{i \in \mathcal{I}_n} F_i(z) A_i(z) \cdot \prod_{j \in \mathcal{J}_n} \tilde{F}_j(z) \tilde{A}_j(z) \,, \tag{11}$$

where, for $0 \leq k < K$, A_k and $\tilde{A}_k$ are expressed as products of all-pass terms (8)

$$A_k(z) = \prod_{p \in \mathcal{P}_k} \left(F_p(z) + \tilde{F}_p(z) \right) , \tag{12}$$

and

$$\tilde{A}_k(z) = \prod_{q \in \mathcal{Q}_k} \left(F_q(z) + \tilde{F}_q(z) \right) . \tag{13}$$

$\mathcal{P}_0$ and $\mathcal{Q}_0$ are two subsets of $\mathcal{K}$ determined at the first 2-band complementary filters of the tree: whenever the filter F_0 (respectively $\tilde{F}_0$) leads to output subband signals with all-pass filters G_n including the same term $F_m + \tilde{F}_m$, this latter is moved right after filter F_0 (respectively $\tilde{F}_0$). This procedure is then repeated to the other 2-band complementary filters ($k > 0$) of the tree until all all-pass filter terms have been moved inside of the tree. For the 2-band complementary filters k, whenever filter F_k leads to output subband signals including the same term in their all-pass filters G_n, this term is moved to F_k. Thus, the subset $\mathcal{P}_k$ corresponds to the intersection of subsets $\mathcal{L}_n$ whose bands involve filtering by filters F_k, i.e.

$$\forall k, \quad 0 \leq k < K, \quad \mathcal{P}_k = \Big(\bigcap_{n | k \in \mathcal{I}_n} \mathcal{L}_n \Big) \setminus \Big(\bigcup_{r<k} \mathcal{P}_r \Big) . \qquad (14)$$

Similarly, subset $\mathcal{Q}_k$ is:

$$\forall k, \quad 0 \leq k < K, \quad \mathcal{Q}_k = \Big(\bigcap_{n | k \in \mathcal{J}_n} \mathcal{L}_n \Big) \setminus \Big(\bigcup_{r<k} \mathcal{Q}_r \Big) . \qquad (15)$$

This procedure minimizes the number of all-pass filter terms which are used, while the transfer function (10) of the original structure before all-pass moving is retained.

IV. Example Filterbank

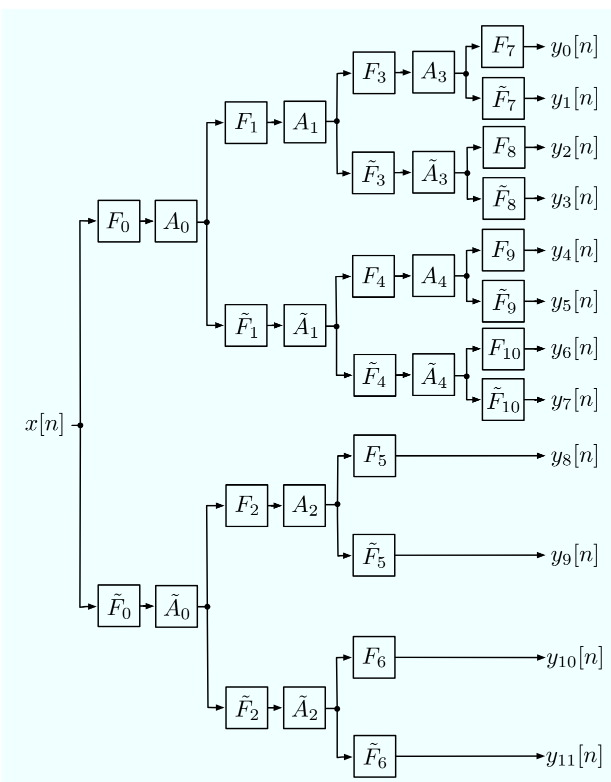

Fig. 3. Example 12-band complementary IIR filterbank.

To illustrate the proposed technique of N-band complementary IIR filterbank design, an example filterbank is described in this section.

A filterbank, approximating an auditory filterbank with 12 bands each having a bandwidth of approximately two times the equivalent rectangular bandwidth (ERB) [9], at a sampling rate of 8 kHz, is considered.

Fig. 3 shows the structure of the example filterbank, where the filters F_k have been designed as Butterworth low-pass filters of order 7 with the cut-off frequencies f_{c_k} defined in Table I.

TABLE I
CUT-OFF FREQUENCIES OF THE EXAMPLE 12-BAND COMPLEMENTARY FILTERBANK.

k	f_{c_k} [Hz]	k	f_{c_k} [Hz]
0	1415	6	3260
1	500	7	125
2	2460	8	370
3	250	9	615
4	800	10	1005
5	1905		

The complementary high-pass filters $\tilde{F}_k$, corresponding to the low-pass filters F_k, are derived using the method described in [8]. The all-pass filters A_k and $\tilde{A}_k$ are derived as explained in Section III, resulting in the subsets $\mathcal{P}_k$ (14) and $\mathcal{Q}_k$ (15) as shown in Table II.

TABLE II
ALL-PASS FILTERS IN THE EXAMPLE 12-BAND COMPLEMENTARY FILTERBANK.

k	$\mathcal{P}_k$	$\mathcal{Q}_k$
0	$\{2, 5, 6\}$	$\{1, 3, 4, 7, 8, 9, 10\}$
1	$\{4, 9, 10\}$	$\{3, 7, 8\}$
2	$\{6\}$	$\{5\}$
3	$\{8\}$	$\{7\}$
4	$\{10\}$	$\{9\}$
$5, 6, 7, 8, 9, 10$	$\emptyset$	$\emptyset$

The frequency responses of the 12 bands of the filterbank are shown in Fig. 4. Note that these magnitude frequency responses do not depend on whether all-pass filters G_n are included or not.

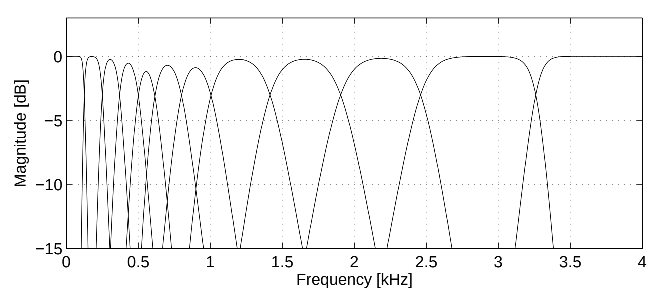

Fig. 4. Magnitude frequency responses of the 12-band example filterbank.

The magnitude frequency response of the sum of all bands is all-pass, as expected, since the filterbank is complementary and as indicated by the bold solid line in Fig. 5. The magnitude frequency response of the sum of bands without all-pass filters is also indicated in the figure as thin solid line, indicating that without the all-pass filters the filterbank is not nearly complementary.

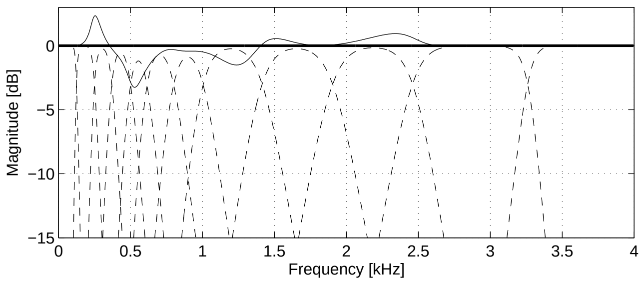

Fig. 5. Magnitude frequency responses of the 12 bands (dashed) and magnitude frequency response of the sum of the 12 bands with (bold solid) and without (thin solid) all-pass filters.

The group delay of the filterbank is shown in Fig. 6, with (bold line) and without (thin line) all-pass filters. Not only the magnitude response, but also the group delay response is improved by using the all-pass filters versus not using them. The group delay with all-pass filters is relatively small and frequency-smooth, as is desired for high audio quality of the reconstructed signal.

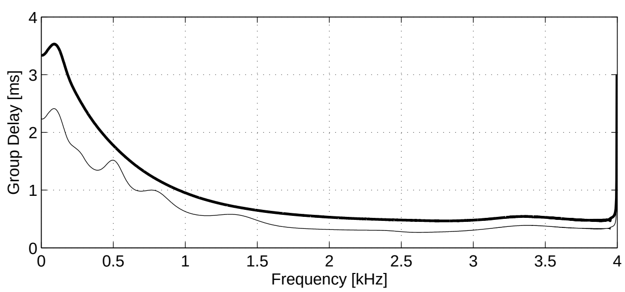

Fig. 6. The group delay of the impulse response of the sum of the bands with (solid) and without (thin) all-pass filters.

V. Discussion

The proposed method allows to build N-band complementary filterbanks with any desired transition frequencies, using a tree structure composed of cascaded 2-band complementary filters and all-pass filters. The specific tree that was used in the example shown in the previous section has been chosen rather arbitrarily. There are many other possibilities for achieving the same frequency decomposition. It has not been assessed in this paper what the differences between different tree structures are for achieving a specific desired frequency decomposition. The differences between using different trees have been smaller than we initially expected.

A non-downsampled complementary filterbank with a frequency invariant group delay has the perfect reconstruction property when the band signals are added to reconstruct the time-domain signal. The value of the time-invariant group delay corresponds to the delay of the filterbank.

The proposed filterbank is complementary, but the group delay is not frequency invariant. Thus, perfect reconstruction is not achieved, but for high audio quality these considerations should be made:

- The relative group delay (difference between minimum and maximum) should be small. A number of papers have discussed the perceptibility of group delay, see e.g. [10], [11]. The data given in Fig. 7 of [10] may imply that the group delay of the described filterbank (bold line in Fig. 6) is such that the reconstructed signal in indistinguishable from the original signal, because the relative group delay of the filterbank is below the given threshold.
- The more frequency-smooth the group delay is, the more is linear-phase property approximated, and the smaller is the perceived difference between the original and reconstructed signal.

VI. Conclusions

The proposed method describes a practical way to implement N-band complementary IIR filterbanks with any desired frequency bands. Such filterbanks are useful for delay-critical audio processing applications. The full-band signal is reconstructed by adding the filterbank bands.

The proposed N-band complementary IIR filterbanks have a tree-like structure, based on simple-to-derive 2-band complementary low-pass and high-pass filter pairs and all-pass filters related to the complementary filters. Filterbanks with any desired band frequencies can be designed, using any tree structure based on 2-band complementary filters with any transition frequencies.

References

[1] J. B. Allen, "Short term spectral analysis, synthesis, and modification by discrete fourier transform," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. 25, pp. 235–238, June 1977.

[2] ——, "Application of the short-time fourier transform to speech processing and spectral analysis," *IEEE ICASSP*, pp. 1012–1015, 1982.

[3] R. E. Crochiere and L. R. Rabiner, *Multirate Digital Signal Processing*, ser. Signal Processing Series. Prentice Hall, 1983.

[4] A. Fettweis, J. A. Nossek, and K. Meerkotter, "Reconstruction of signals after filtering and sampling rate reduction," *IEEE Trans. Acoustics, Speech, Signal Processing*, vol. 33, pp. 893–902, August 1985.

[5] P. A. Regalia, S. K. Mitra, P. P. Vaidyanathan, M. K. Renfors, and Y. Neuvo, "Tree-structured complementary filter banks using all-pass sections," *IEEE Trans. Circuits Syst.*, vol. 34, pp. 1470–1484, December 1987.

[6] Y. Neuvo and S. K. Mitra, "Complementary iir digital filters," in *Proc. IEEE Int. Symp. on Circuits and Systems*, May 1984, pp. 234–237.

[7] T. Samaräki, "On the design of digital filters as a sum of two all-pass filters," *Proc. IEEE Int. Symp. on Circuits and Systems*, vol. 32, pp. 1191–1193, Novenber 1985.

[8] P. P. Vaidyanathan, S. K. Mitra, and Y. Neuvo, "A new approach to the realization of low-sensitivity iir digital filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 34, pp. 350–361, April 1986.

[9] B. R. Glasberg and B. C. J. Moore, "Derivation of auditory filter shapes from notched-noise data," *Hear. Res.*, vol. 47, pp. 103–138, 1990.

[10] J. Blauert and P. Laws, "Group delay distortions in electroacoustical systems," *The Journal of the Acoustical Society of America*, vol. 63, p. 1478, 1978.

[11] S. Flanagan, B. Moore, and M. Stone, "Discrimination of group delay in clicklike signals presented via headphones and loudspeakers," *Journal of the Audio Engineering Society*, vol. 53, no. 7-8, pp. 593–611, 2005.