NONLINEAR ADAPTIVE IMAGE PROCESSING IN TRANSFORM DOMAIN

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ABSTRACT

A family of local adaptive filters for image restoration and enhancement is described. The filters work in a moving window in the domain of an orthogonal transform and, in each position of the window, produce an estimation of the central pixel of the window by nonlinear transformations of the window spectral coefficients. The filter synthesis on the base of local mean squared error restoration criterion is outlined, the use of Discrete Fourier and Discrete Cosine Transforms as orthogonal transforms is justified and recursive algorithm of multicomponent local DCT spectral analysis is described. Performance of filtering is illustrated by examples of edge preserving noise suppression and blind restoration of color images

1. INTRODUCTION

Image processing in transform domain rather than in signal domain suggests certain advantages in terms of convenience of incorporating a priory knowledge on images into the design of processing algorithms and in terms of computational expenses. The transfer from signal domain into the transform domain is especially promising if it is applied locally rather than globally. This has already been successfully demonstrated by transform image coding methods. In this paper, we outline a family of nonlinear local adaptive filters for image denoising, restoration and enhancement that work in a transform domain in a moving window. In each position of the window, the filters estimate spectrum of the image fragment within the window in the chosen transform basis, nonlinearly modify the spectral coefficients and then perform inverse transform to obtain an estimate of the central pixel of the window. The synthesis of the filters is based on local criteria of image processing quality ([6-9]). It will be shown also that, thanks to the existence of recursive algorithms for computing local image spectrum in a moving window, the computational complexity of the algorithms is *O(Window size)* which is the lower bound for the computational complexity of space-variant filtering. The filters are illustrated by examples of edge preserving color image denoising and blind image restoration.

2. MULTI COMPONENT LOCAL ADAPTIVE FILTERS

2.1. Optimal local adaptive filters

We rest our approach upon local criteria ([1, 4]) that evaluate image processing quality for each particular image sample as an average, over a certain neighborhood of the pixel, of a certain loss function that measures deviation of the estimation of each pixel in the neighborhood from its true value. For multi component images, if the neighborhood is defined as a spatial one and the loss function is a squared difference between the estimation and the true value of the pixel, the local criterium requires minimization of the functional:

$$AVLOSS\,(k_1, k_2) = \mathbf{AV}_{\text{imgsys}} \left\{ \sum_{c=1}^{C} \sum_{\substack{|k_1 - n_1| \le M_1 \\ |k_2 - n_2| \le M_2}} \left| \widehat{a}^{(c)}_{n_1,n_2} - a^{(c)}_{n_1,n_2} \right|^2 \right\} \tag{1}$$

where $\left\{ a^{(c)}_{n_1,n_2} \right\}$ and $\left\{ \widehat{a}^{(c)}_{n_1,n_2} \right\}$ are samples of ideal image components and their estimations, respectively, C is the number of image components and (k_1, k_2) are running coordinates in the image plane.

The design of local adaptive filters is aimed at finding a mapping of the set of observed image samples $\left\{ b^{(c)}_{n_1,n_2} \right\}$ to the set of estimations $\left\{ \widehat{a}^{(c)}_{n_1,n_2} \right\}$ that provides minimum to the criterium for each pixel of the image. To make the design constructive, one should parameterize the mapping. One of the way to do this is to assume that the estimation of each pixel is obtained as a weighted sum of the observed pixels in its neighborhood.

For local adaptive filters, filter coefficients have to be determined in each particular position of the running window from the observed image samples within the window on the base of a priori information regarding images under processing. While formulation of this information in the image domain is apparently problematic, it is much simplified in the domain of certain orthogonal transforms, such as DFT and DCT because power spectra of image fragments in such transforms exhibit much more regular behavior than that of pixels themselves. Note that precisely this fact motivates and justifies also fragment wise transform image coding and explains its efficiency.

Therefore we will assume in what follows filtering in a transform domain and will design filters that operate, in each position (k_1, k_2) of the window, in the following 3 steps:

1. Computing spectral coefficients $\left\{\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\} = \mathbf{T}\mathbf{b}^{(k_1,k_2)}$ of the observed image fragment $\mathbf{b}^{(k_1,k_2)}$ within the window over the chosen orthogonal transform $\mathbf{T}$.

2. Multiplication of the obtained spectral coefficients by the filter coefficients $\left\{\eta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\}$:

$$\widehat{\alpha}_{r_1,r_2,\sigma}^{(k_1,k_2)} = \eta_{r_1,r_2,\sigma}^{(k_1,k_2)} \beta_{r_1,r_2,\sigma}^{(k_1,k_2)} . \tag{2}$$

3. Inverse transformation $\mathbf{T}^{-1}$ of the output signal spectral coefficients $\left\{\widehat{\alpha}_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\}$.

Here, superscripts $(r_1, r_{2,}\sigma)$ are corresponding indices in the transform domain.

With this approach, the synthesis of local adaptive filters is reduced to the determination of $(2M_1+1)(2M_2+1)C$ filter coefficients $\left\{\eta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\}$. For the optimal filter design in the domain of an orthogonal transform, one can, by virtue of the Parceval's relation, reformulate the criterion of Eq.(1) in terms of signal spectra :

$$AVLOSS\,(k_1,k_2) = AV_{imsys}\left\{\sum_{r_1,r_2}\sum_{\sigma=1}^{C}\left|\eta_{r_1,r_2,\sigma}^{(k_1,k_2)}\beta_{r_1,r_2,\sigma}^{(k_1,k_2)} - \alpha_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right\} \tag{3}$$

By minimizing $AVLOSS\,(k_1,k_2)$ with respect to $\left\{\eta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\}$ one can find that the optimal values of the coefficients of the filter that minimizes the filtration error as defined by Eq.(1) may be found from the following equation:

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \frac{AV_{imsys}\left\{\alpha_{n_1,n_2,\sigma}^{(k_1,k_2)}\left(\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right)^{\star}\right\}}{AV_{imsys}\left\{\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right\}} \tag{4}$$

with $\star$ denoting complex conjugate. The design of the local adaptive filter of Eq.(4) is therefore reduced to an estimation of local power spectrum of the input image fragment and its mutual local spectrum with an "ideal' image.

2.2. Local adaptive filters for image restoration

Assume that image distortions can be modeled by the equation:

$$\mathbf{b} = \mathbf{L}\mathbf{a} + \mathbf{n}, \tag{5}$$

where $\mathbf{L}$ is a linear operator of the imaging system and $\mathbf{n}$ is a random noise. Assume also that the imaging system operator $\mathbf{L}$ is such that the distorted image can be described in the domain of the chosen orthogonal transform by the following relationship:

$$\beta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \lambda_{r_1,r_2,\sigma}^{(k_1,k_2)}\beta_{r_1,r_2,\sigma}^{(k_1,k_2)} + \nu_{r_1,r_2,\sigma}^{(k_1,k_2)} \tag{6}$$

where $\left\{\lambda_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\}$ are running representation coefficients of the linear operator $\mathbf{L}$ in the domain of the orthogonal transform and $\left\{\nu_{r_1,r_2,\sigma}^{(k_1,k_2)}\right\}$ are zero mean spectral coefficients of the realization of the noise interference. Then one can obtain from Eq.(4) that optimal restoration filter coefficients are defined as:

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \frac{\left|\lambda_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\left|\alpha_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2}{\lambda_{r_1,r_2,\sigma}^{(k_1,k_2)}AV_{imsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)} \tag{7}$$

Filter of Eq.(7) can be regarded as an "empirical" Wiener filter that assumes estimation of the parameters involved in its design locally from the observed distorted image fragments.

2.3. Local adaptive filters for image enhancement

Image enhancement is a processing aimed at assisting visual image analysis. A reasonable basis for the design of linear filter parameters for image enhancement is an assumption that the filtering should "restore" a "useful" signal, that is image details to be enhanced or extracted for the end user's convenience, against a "noise" background that obscures detail interpretation. Following the above approach, one can, by analogy with the derivation of Eq.(7) design a filter that provides minimum to the average squared modulus of the difference between this useful signal and the filtered signal:

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \frac{AV_{obj}\left(\left|\alpha_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)}{AV_{obj}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)} \tag{8}$$

where AV_{obj} denotes averaging over such variations of the useful object(s) as object position, size, shape, etc.

Another useful criterion for image enhancement is the criterion of signal spectrum restoration ([5]) that requires restoration of power spectrum of the object signal. For this criterion, the following obvious filter results:

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \left(\frac{AV_{obj}\left(\left|\alpha_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)}{\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)} \right)^{1/2} \qquad (9)$$

Finally, one may desire to maximize the ratio of the signal derived for the desired object to the standard deviation of the image's background component at the filter output. The following filter is the result:

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \frac{\left(\alpha_{r_1,r_2,\sigma}^{(k_1,k_2)}\right)^{\star}}{AV_{obj}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)}. \qquad (10)$$

3. TRANSFORM SELECTION

The selection of orthogonal transforms for the implementation of the filters is governed by the convenience of formulating a priori knowledge regarding image spectra in the chosen base, by the accuracy of spectra estimation from the observed data that is required for the filter design, and by the computational complexity of the filter implementation.

We believe that the selection of the Discrete Fourier and Discrete Cosine Transforms is the most appropriate in all these respects. It is commonly known that spatial power spectra of images in the base of DFT and DCT exhibit a regular feature of decaying with the rise of spatial frequencies. DCT is advantageous to DFT in terms of the accuracy of spectral estimation due to the fact that DCT, being DFT of signals evenly extended outside their borders, substantially eliminates boundary effects that are characteristic for the DFT proper because of its periodicity. Both, DFT and DCT in a running window can be computed recursively ([4],[6],[7]). Therefore, the computational complexity of filtering in the base of DFT and DCT in a running window is of the order of O(Size of the window) operations which is the theoretical minimum for a general space variant filtering. As for the component-wise transform, if we assume that it is separable of the transforms over spatial coordinates, it will not affect the computational complexity. We will not specify it here and leave the question of the selection of it open.

The recursive algorithm of 1-D DCT is decribed in Refs.([3],[4]). The extension of this algorithm to the recursive computation of 2-D DCT and the 3-D combined transform along coordinate k_1 is straightforward thanks to the separability the transforms:

$$\alpha_{r_1,r_2,\sigma}^{(k_1+1,k_2)} = Real\left(\widetilde{\alpha}_{r_1,r_2,\sigma}^{(k_1+1,k_2)}\right) = Real\left\{ \widetilde{\alpha}_{r_1,r_2,\sigma}^{(k_1,k_2)} \exp\left(-i\pi\frac{r_1}{2M_1+1}\right) + \left[(-1)^{r_1}\widetilde{\alpha}_{r_2,\sigma}^{(k_1+M_1+1,k_2)} - \widetilde{\alpha}_{r_2,\sigma}^{(k_1-M_1,k_2)}\right] \times \exp\left[-i\pi\frac{r_1}{2(2M_1+1)}\right] \right\}. \qquad (11)$$

where $\left\{\widetilde{\alpha}_{r_2,\sigma}^{(k_1+M_1+1,k_2)}\right\}$ and $\left\{\widetilde{\alpha}_{r_2,\sigma}^{(k_1-M_1,k_2)}\right\}$ are 2-D auxiliary spectra of the window columns that, respectively, come in and come out of the window at its shift from k_1-th to (k_1+1)-th pixel in the current row k_2:

$$\widetilde{\alpha}_{r_2,\sigma}^{(k_1-M_1,k_2)} = \sum_{n_2=k_2-M_2}^{k_2+M_2} \sum_{c=0}^{C-1} a_{k_1-M_1,n_2}^{(c)} \times \exp\left[i\pi\frac{(n_2-k_2+M_2+1/2)}{2M_2+1}r_2\right]\varphi_c(\sigma), \qquad (12)$$

and $\{\varphi_c(\sigma)\}$ are basis functions of the component-wise transform.

Note that when the window shifts one pixel down the coordinate k_2 these 2-D auxiliary spectra can in their turn be computed recursively.

4. FILTER IMPLEMENTATION: LOCAL ADAPTIVE FILTERS WITH NONLINEAR PROCESSING IN TRANSFORM DOMAIN

The main issue in the design and implementation of local adaptive filters in the transform domain is the estimation of averaged spectra of the observed image fragments, of the desired signal and of the component regarded as an additive signal independent noise or as a background. In this estimation, *a priori* knowledge regarding image spectra should be formulated. Noise power spectrum $\mathrm{AV}_{imsys}\left(\left|\nu_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)$, in image restoration can either be known from the imaging system design or can be estimated from observed noisy images ([4]). The estimation of the observed image fragment spectrum $AV_{imsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)$ may be carried out by one or another smoothing the observed spectrum of the fragment being processed. The estimation of the spectrum of "ideal" image fragments in the case of additive signal-independent noise can then be carried out using the relationship:

$$\left|\lambda_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2 \left|\alpha_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2 \simeq \max\left\{0,\ AV_{imgsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2 - \left|\nu_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)\right\}, \qquad (13)$$

In this way we arrive at the following implementation of filters for image denoising and deblurring:
if $\lambda_{r_1,r_2,\sigma}^{(k_1,k_2)} \neq 0$,

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = \max\{0,$$

$$\left.\frac{AV_{imsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right) - AV_{imsys}\left(\left|\nu_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)}{\lambda_{r_1,r_2,\sigma}^{(k_1,k_2)} AV_{imsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right)}\right\}; \quad (14)$$

otherwise, $\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = 0$. Similar implementations can be obtained for filters of Eqs.(8-10) for image enhancement as well. One can further generalize or/and simplify these transformations. The two following filter modifications can be suggested:
a "rejective" filter

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = 1/\lambda_{r_1,r_2,\sigma}, \quad (15)$$

if $AV_{imsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right) \geq thr$ and $\lambda_{r_1,r_2,\sigma} \neq 0$; and

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = 0,$$

otherwise, where the value of thr is associated with the variance of additive noise, and
a "fractional spectrum" filter

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = G\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^{P-1} \quad (16)$$

if $AV_{imsys}\left(\left|\beta_{r_1,r_2,\sigma}^{(k_1,k_2)}\right|^2\right) \geq thr_\sigma$; and

$$\eta_{r_1,r_2,\sigma}^{(k_1,k_2)} = 0,$$

otherwise, with P as a spectrum enhancement parameter and G as an energy normalization parameter. When $P = 1$, the filter is equivalent to that of Eq. 15. The selection $P \leq 1$ results in signal energy redistribution in favour of weaker (most frequently, higher frequency) components of local spectra. This modification is useful for image blind deblurring and image enhancement.

Figs.1 and 2 illustrate edge preserving noise suppression and blind deblurring capability of the above filters in the processing of color images. In these experiments, DCT was also used as the component-wise transform.

5. CONCLUSION

Local adaptive filters that work in a moving window in the domain of an orthogonal transform and perform image filtering by means of a nonlinear modification of image local spectral coefficients have been described. The filters are designed on the base of local root mean squared error criterion. DFT and DCT are recommended for as the transforms thanks to the simplicity of formulating of a priori knowledge regarding images in terms of their DFT/DCT spectra and to the existence of recursive algorithms for local spectral analysis in the base of DFT and DCT. Such orthogonal transforms as Haar Transform, and Hadamard (Walsh) transform as well as wavelet transforms can also be implemented recursively and, therefore, are potentially applicable for the described filtering.

6. REFERENCES

[1] L. P. Yaroslavsky, "Linear and Rank Adaptive Filters for Image Processing", in Digital Image Processing and Computer Graphics. Theory and Applications, L. Dimitrov and E. Wenger, Ed., Wien, Muenchen: R. Oldenburg, pp. 374, 1991.

[2] L.P. Yaroslavsky, Local Adaptive Filters for Image Restoration and Enhancement, In: 12^{th} Int. Conf. on Analysis and Optimization of Systems. Images, Wavelets and PDE's, Paris, June 26-28, 1996, Eds.: M.-O. Berger, R. Deriche, I. Herlin, J. Jaffre, J.-M. Morel, Springer Verlag, Lecture Notes in Control and Information Sciences, 219., 1996, pp. 31-39.

[3] L. Yaroslavsky, Local Adaptive Image Restoration and Enhancement with the Use of DFT and DCT in a Running Window, In: Wavelet Applications in Signal and Image Processing IV, Eds.: M. A. Unser, A. Aldroubi, A.F. Laine, SPIE vol. 2825, pp. 2-11.

[4] L. Yaroslavsky and M. Eden, Fundamentals of Digital Optics, Birkhauser, Boston, 1996.

[5] W. K. Pratt, Digital Image Processing, Wiley, N. Y., 1978

[6] L. Yaroslavsky, Parallel and Recursive Implementation of Digital Filters, Radio Engineering, v. 39, 1984, pp. 71-75

[7] R. Y. Vitkus and L.P. Yaroslavsky, "Recursive Algorithms for Local Adaptive Linear Filtration", in Mathematical Research., Eds.:L.P. Yaroslavskii, A. Rosenfeld and W. Wilhelmi, Berlin: Academy Verlag, pp. 34-39, 1987.

Figure 1: Noisy RGB image components (upper raw) and filtered image component (bottom raw). Standard deviation of noise in each of 3 image component was 20 with the signal range 0-255. Filtering was performed by filter of Eq. (15) with $\lambda_{r_1,r_2,\sigma} = 1$.

Figure 2: Blind image restoration: RGB components of the initial image (upper raw) and corresponding components of the restored image (botom raw). The filtering was performed by the filter of Eq. (16) with $P = 0.75, G = 2$ and $thr_r = 50; thr_g = 30$ and $thr_b = 60$.